

Contemporary Advances in Physics—XIII. Ferromagnetism

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Magnetism was revealed to Europeans by pieces of a mineral later to be called lodestone, which lay scattered in the fields of Magnesia in Asia Minor, and were endowed with the curious power of attracting iron. They who first noticed it were apparently Greeks of the period before the practice of writing; for legends of the discovery were transmitted by the Greeks of later centuries, legends entangled with tales of Cretan shepherds and the myth of Medea. Electricity was disclosed, evidently in the same dim period and region, by fragments of amber on which friction conferred the remarkable power of attracting shreds and flakes of light materials.

By these quaint phenomena electricity and magnetism were disclosed to the European world before the beginnings of written history; and the intimations were recorded in writings of classical antiquity, and handed down from generation to generation. Yet two millennia and more were destined to flow past, before sufficiently many further data should be gathered to make possible the forming of a valid conception of either. The nineteenth century arrived, before anyone detected the signs that the two are but different aspects of one fundamental entity. Obviously the early hints were not sufficient; but it would not be well to conclude that therefore the Greeks were unwise. If they are indicted for stupidity because they did not understand the lodestone and the electrified amber, the indictment lies also against ourselves. For these are instances of ferromagnetism and of frictional electricity; which is to say, they belong to provinces which to this day are not fully incorporated into the empire of the theory of electricity and magnetism.

How then does it happen that the phenomena earliest discovered must still be listed among the least well understood? There is nothing unusual in this. There is no general reason for expecting that the phenomena which occur spontaneously and frequently and conspicuously in Nature should be the easiest to understand. On the contrary, it frequently happens that they are much less instructive and interpretable than others which can be brought to pass only by careful choice of conditions and skilful experimentation. The history of physics abounds in instances of such contrasts, and there is none more striking than the one with which I am to deal. Many phenomena of

magnetism are well explained by the contemporary theory, many seem admirably clear; but none of these was or could have been witnessed by the Greeks. We know much about the magnetic properties of gases, dilute solutions, free atoms, elements and compounds which are so feebly magnetizable that before 1830 they were not supposed to be "magnetic" at all; we are still perplexed by the behaviour of iron and lodestone. This is the reason why there are textbooks of magnetism, in which hundreds of pages are devoted to the data and the theories of a number of effects most difficult to perceive and known to none but physicists, while the magnets of daily experience are dismissed with a chapter or two of mere description. As for electrified amber and its kindred, they are fortunate to have a few paragraphs of any modern treatise on electricity bestowed upon them.

Frictional electricity is not a very striking phenomenon, nor is it valuable in engineering; consequently it has been allowed to slip into obscurity, shunned by cautious students on the hunt for problems promising immediate returns. Ferromagnetism is not so unobtrusive. Much of the electric machinery which has transformed the world since Napoleon derives all its efficacy from certain blocks of iron or magnetizable alloy, enmeshed among the wires. So useful a property of matter does not consent to lie neglected; physicists are forced to hearken to its insistent demands for attention. Ambition to achieve some technical advance supplies a strong incentive; and there is a feeling of humiliation that a quality of matter so conspicuous and so remarkable, and so remarkably limited to a particular class of substances not in other ways exceptional, should not be properly connected with the structure of contemporary physics. For these and other motives, there are always physicists engaged in the struggle with the problem of ferromagnetism—no mean struggle, for the difficulties are truly serious. It was a tough problem which was offered to the Greeks and which they rejected, when they saw the lodestone, took note of it, and left it for the modern world to study.

Some of the difficulties of ferromagnetism may be peculiar to it. Others, it is to be feared, are examples of the troubles which are reserved for scientists by the internal properties of solid bodies generally, and which physicists will some day be forced to confront when the obvious problems of gases and free atoms are exhausted, if they are not sooner incited by curiosity or by the requirements of engineering. Most of the great conquests of recent physics have been achieved through the study of gases, or of those properties of matter which are the same for the solid as for the gaseous state. It is but natural to

wish to postpone as long as possible the attack upon the intrinsic properties of solids; but there is no evading it in the study of ferromagnetism, for this is a property of solids only, and not even of transparent solids at that. One would wish at least to be permitted to restrict the study to pure elements or simple compounds; but many of the most interesting of the ferromagnetics belong among those bewildering substances the alloys, which form what the mathematicians describe as a *continuum* beside and among the great yet finite number of chemical compounds. If one were to work only with perfectly pure iron (supposing that one could get such a substance, or could recognize it when he had it!) the problem would not yet be simple; for every species of mechanical and thermal treatment, and magnetization itself, would transform the iron into a new sort of solid.

These difficulties I will stress in the pages of this article. There is another. The information about a ferromagnetic substance—the prime material required for theorizing or for practical applications—is usually furnished in the form of so-called *I*-vs-*H* curves; that is to say, relations between the “intensity of magnetization” and the “magnetizing field.” These curves play the part of the ultimate data of experience. Yet they are not ultimate data; the “magnetizing field” is seldom actually measured, the “intensity of magnetization” almost never. These entities *I* and *H* are deduced from experience by means of a theory. The theory is indispensable. If an uninstructed person were presented with a number of variously-shaped pieces of iron, and a battery and a coil of wire with which to produce any desired magnetic field, and any number of measuring-instruments, he would find it extremely hard to select something to measure that might yield a coherent and intelligible set of data. He would be able to show in a vague way that the greater the magnetic field acting upon any piece of iron, the more powerful a magnet it becomes; but if he were to search for some precise measurable quantity that could serve as a measure of the power of the magnet, and that would be *characteristic of iron as a substance and not merely characteristic of individual pieces of iron as individuals*, his search would be a long one. From what I have just called “the theory” he would find out what to measure, and how to calculate from it the value of something characteristic of iron and not affected by the shape of the pieces; he would find out how to trace an “*I*-vs-*H*” curve. This curve would serve in turn as a basis for theories of ferromagnetism; but theory would have entered already into the preparation of the curve. I shall therefore devote the first section of this article to the principles according to which such curves are determined from the immediate data. Any

reader who feels that these principles are familiar, or self-evident, or unimportant, may leap to the second section and the third, in which the I -vs- H curves are accepted as the data of experience.

I shall not venture a definition of ferromagnetism until nearly the end of the article. Such a definition is not easy to make, unless one takes refuge in the statement that "ferromagnetism is the kind of magnetism displayed by iron." I can only regret the frequency with which such ponderous words as *ferromagnetism* and *permeability* and *susceptibility* and *magnetization* and *magnetostriction* must needs appear. The subject is encumbered by its heavy vocabulary; it ought to have a new one made up entirely of short and vivid words.

A. ANALYSIS OF THE MAGNETIZATION OF MAGNETIZED BODIES.

Let us imagine a collection of magnets such as one frequently sees, horseshoe magnets for example, with their ends painted red and blue. We know that (if the painting was done properly) the red end of each attracts the blue ends and repels the red ends of the others; the blue end attracts those which the red end repels and repels those which the red end attracts. It seems as if the ends of the magnets were covered with invisible substances—one kind on all the red ends, the other on all the blue ends—so constituted, that a sample of either substance attracts all samples of the other sort, repels all samples like itself. Coulomb found that if the magnets were long and slender, so that the power of attracting and repelling was concentrated very closely about the extremities of each, these extremities attracted or repelled one another according to an inverse-square law. That suggested gravitation and electric force; which suggested in turn that, even as matter is the source of gravitation and electric charge is the source of electric force, so also there is an invisible thing called magnetism which inhabits iron—or rather, two invisible things, positive magnetism and negative magnetism, which may be pulled and pushed around inside and over the surface of a piece of iron. This notion of a pair of invisible and mobile fluids is very helpful, and I shall use it in several passages; yet the reader must not take it as corresponding to the actual reality. We cannot imagine two or even one perfectly mobile magnetic fluid, for a well-known reason.

The reason is, that even though a magnet may appear to carry nothing but positive magnetism on one of its ends and nothing but negative magnetism on the other, yet it is not possible to cut off anywhere a piece containing only one of these kinds. In fact it is not possible anywhere to cut off a piece not containing equal quantities of the two kinds of magnetism. Any piece of matter always contains as much

positive magnetism as negative; so also does any smaller fragment broken off from the piece, and any still smaller bit broken out of the fragment, and so forth until the original piece is crumbled into dust, each particle of which still contains as much magnetism of either sign as of the other.*

Now this requires that when we subdivide a magnetized piece of iron into tiny parcels or volume-elements, not by the hammer nor the file but by the exercise of the imagination, these volume-elements must themselves be imagined as magnets each invested with a positive pole and a negative pole and a magnetic axis pointing in some particular direction. I am not implying atoms by these "parcels"—we shall as yet have nothing to do with atoms. The process of dividing a substance into imaginary small volume-elements has nothing in common with the construction of atoms or atom-models; quite the contrary! It is a process which every physicist undertakes, whenever he desires to analyze the flow of water or the vibrations of air or the strain of a twisted rod or any of a multitude of problems concerning pieces of matter, which, whatever his views about atoms, he intends to regard as continuous media for the nonce. Well! in dealing with magnetism, it is not sufficient to conceive these volume-elements as cubical or otherwise-shaped bits of matter entirely uniform and isotropic in their qualities; they must be conceived as being little magnets themselves.

This is the reason why we are taught to imagine a piece of magnetized iron as a collection of tiny cubes, each bearing positive magnetism spread like a coat of paint over one side, and negative magnetism over the side opposite; or as a bundle of filaments which, where they come out to the surface of the piece, divide it into a pattern of area-elements each of which is overspread with magnetism positive or negative; or as a pile of laminæ, somewhat like a nest of saucers, each of which is covered with magnetism of the two signs on its two sides. This is the reason why, developing the first of these conceptions (which contains implicitly the other two), we are taught to picture a function called the *intensity of magnetization*, which has a definite value at each point within the magnet, and may be visualized with the aid of the imaginary cubes. Select a point in the interior of the magnet, and imagine it surrounded by a cubical volume-element of thickness d and face-area d^2 and volume d^3 ; and imagine two opposite sides of the cube to be covered with magnetism of opposite signs painted on with a surface-density I , so that each side bears a quantity Q which

* The best evidence for this statement is the fact that magnets in a uniform magnetic field such as that of the earth experience no force tending to displace them bodily though they experience a torque tending to orient them.

is Id^2 . This cube would be a minute magnet having the moment* Qd which is Id^3 , directed normally to the two sides coated with magnetism; for I is a quantity possessing both magnitude and direction, a vector quantity and not a scalar—this is a way of expressing the complexity to which an allusion was made in the last paragraph. The piece of magnetized material is to be visualized as the assembly of all these little cubes, each having a magnetic moment equal to its volume multiplied into the value of I prevailing in it. The force exerted by the piece anywhere outside of its volume is to be considered as the sum of the forces there exerted by all the little magnets. The entity I plays the rôle of a magnetic-moment-per-unit-volume. It is this entity which is defined as the *intensity of magnetization* of the material.

This, it may be objected, is something quite unverifiable; for one cannot penetrate into the interior of a piece of iron, and find out whether it contains such an entity as this vector I . Quite so! and this is another of the great difficulties in ferromagnetism, though not peculiar to ferromagnetism alone, for it besets in greater or less degree every problem of the properties of solid bodies. The state of affairs within a piece of magnetized iron is the leading problem of ferromagnetism, indeed it is the one problem which contains all the rest. But there is no way of ascertaining that state of affairs, for there is no way of putting a measuring-instrument into a piece of iron. One might scoop a hole in the iron to make a place for the magnetometer, but then the magnetometer would be in the hole and not in the iron. The field of magnetic force outside the magnet can be plotted, the lines of force in the field can be followed up to the very edge of the magnetized material, but there they dive and they disappear. When one sees a sketch of a magnet and its environment, in which the lines of force coming up from all sides to the surface of the magnet are connected in pairs by "lines of induction" passing through the body of the magnet, he should realize that while the lines of force outside are a map of a field which can be explored, the lines of induction within are hypothetical altogether.

Why then take the trouble of conceiving entities such as these, intensity of magnetization I and induction B , since they are solely imagined to exist in a locality where there is no possible means of penetrating to seek them? The reason is this, and this only: Confined though they are within the bodies of the magnets, they facilitate the

* "Magnetic moment" is usually defined by inviting the reader to imagine a magnet so long and slender that its "magnetism" is concentrated almost completely at its ends or "poles"; the moment of such a magnet is the product of its length into the amount of magnetism, or "polestrength," at either end. Actual magnets have no true poles. The moment of an actual magnet is the torque which a unit field exerts upon it when it is normal to the direction in which the field tends to set it.

understanding of the effects which the magnets produce outside. Induction and intensity of magnetization are things which are supposed to exist *inside* a solid magnetic body, to make it possible to predict what effects that body produces in the world outside of itself—the only region which can be entered with or without measuring-instruments.

Now if a magnet were delivered over by Nature in fixed and permanent state, so that nothing which could be done to it would alter its behavior towards surrounding objects, the problem of determining I would be relatively simple. It would amount to this: to build up a structure of little cubical magnets occupying the same volume as the actual magnet, and producing everywhere outside that volume the same field as the actual magnet is observed to produce. In other words, it would consist in seeking a function I of the coordinates x , y , z of the points within the volume of the magnet, fulfilling the following condition: when this volume is subdivided into small cells of volume dv , and each is treated as a magnet of moment $I dv$, and the forces exerted by all these little magnets at any point outside of the volume are summed together, their sum shall turn out to be the same as the force which the actual magnet is observed to exert at that point.

This however is not the whole of the actual problem. The force which a magnet exerts at any particular point in its vicinity depends upon the magnetic fields which are impressed upon it by external objects—other magnets, or electric currents, or the earth itself. It becomes a different magnet when it is subjected to a different field. The process of finding a function I fulfilling the condition made above must therefore be carried through anew whenever the exterior fields acting upon the magnet are changed.

This variability makes the problem much more difficult. Yet in some cases it can be dealt with, in the same manner as the more restricted problem of analyzing an unchanging magnet into volume-elements; and in dealing with it, the first foundations of a theory of magnetism are laid down.

A piece of iron is observed to become a different magnet, whenever the impressed magnetic field is changed. Very well! we will try to describe the difference, by assuming that each of the volume-elements into which we have mentally divided the piece becomes itself a different magnet. The change in the magnetism of the piece is all too likely to be complicated and obscure; but we will simplify by supposing that the magnetization of each of the volume-elements depends upon the magnetic field prevailing in it, according to some law which is

the same for all the volume-elements; that there is a fixed relation between the intensity of magnetization at a point and the field existing at that point, which is the same everywhere within the supposedly uniform piece of iron, which is a quality of that particular kind of iron. If there is no such relation, the whole procedure is likely to be futile. If there is such a relation, it is the fundamental fact of magnetism; and the first business of the student of magnetism is to determine it for as many substances, under as many conditions, as he can. We shall presently see that most research in ferromagnetism is devoted to determining this relation, by methods which would not yield self-consistent results, did it not exist.

But we shall attain nothing by merely assuming that there is such a relation, unless we make another assumption concerning the field prevailing within the magnet; for it is quite inaccessible, we cannot enter in to measure it. Let us therefore suppose that the field produced at any point inside the magnet, by the objects outside—be they laboratory magnets, or electric currents, or the earth itself—is the same as they would produce at that point were the magnet taken away, leaving them the same. The outer parts of the magnet are supposed *not* to shield the inner parts from the magnetic influences of the outer world. This is a natural corollary of the supposition we have tacitly made already, that the outer volume-elements of the magnet do not shield the outer world from the magnetic forces due to the inner volume-elements. We assume it; and we assume that the intensity of magnetization and the magnetic field, the vectors I and H , are parallel to one another,* and that there is a relation between their magnitudes which is the same for every point within the magnet.

On proceeding to test this set of assumptions by the appeal to experiment, we encounter results which at first sight seem to destroy them. For instance, let us immerse a short rod of iron (quite demagnetized to begin with) in the uniform magnetic field produced within a long cylindrical tube by an electric current flowing through a coil of wire, a solenoid, evenly wrapped around the tube. The field H_e which the current would produce within the tube were the iron not there is uniform in magnitude and direction, everywhere parallel to the axis of the solenoid. By the last assumption, this is the field which the current produces everywhere inside the iron. We map the magnetic field produced by the rod in its vicinity, and determine

* There are cases, neither few nor trivial, in which I and H cannot always be supposed parallel; for instance, when the magnet is a large crystal, or when it is a plate of metal which has been cold-rolled, or when the direction of the magnetizing field is changed after the substance is already perceptibly magnetized. But if I were to expound the most general actual case, this article would never come to an end.

the function I which describes the magnetization which would produce such a field. The vector I is not uniform throughout the iron, either in direction or in magnitude. Though H_e is the same everywhere within the metal, I varies from point to point. This result by itself seems to demolish the assumptions.

The contradiction however is only apparent; it vanishes if we make due allowance for the field produced at every part of the magnet by the other parts, for *the effect of the magnet upon itself*. Continuing to use the illustration of the short rod in the uniform impressed field: the distribution of elementary magnets which the function I expresses, and which produces at every point outside the iron a calculable field agreeing with the field there observed, should also produce a calculable field at every point within the iron. Considering that we have assumed that the force due to even the innermost volume-element of the magnet is exerted unimpeded everywhere in the outside world, we cannot consistently avoid assuming that its force is exerted unimpeded upon the other volume-elements as well. Thus it is reasonable to suppose that if the value of I at any point in the iron is controlled by the magnetic field there prevailing, then the truly controlling field comprises not only the one (H_e) due to the external agencies, but also the other (H_i) due to the multitude of little magnets presumed to constitute the piece of iron. The value of I should depend on the resultant H of H_e and H_i . In the present case of the short rod inside the solenoid, the vector H_e is uniform, but the vector H_i varies from point to point, and consequently so does the resultant H of H_e and H_i , and consequently so does I . More properly, I should not use such a word as "consequently" at all; both I and H vary from point to point, either accounting for the other, either being cause and either being effect.

This, by the way, is one of the reasons why as a rule it is not possible to analyze the magnetization of a magnet by cutting it into little pieces and measuring the moment of each separately. When such a piece is isolated from the rest of the magnet, the field acting upon it is changed even though all the external field-producing agencies remain the same. The other reason for not cutting up a magnet is, that the stresses exerted on the material in the process of cutting are likely to change it into some very different ferromagnetic material—but of this, more later.

The problem of determining I now assumes its full scope. For every magnet, or let us say for every piece of magnetized iron, there should be a function I describing its magnetization, defined at every point within it and satisfying these conditions:

First, it should account for the field due to the magnet at every point outside;

Second, its value at every point inside the magnet should be a definite function of the thing which we have just tentatively defined as "the magnetic field" at that point; *viz.* the resultant of that field which the external agencies would produce were the magnet away, and that which the magnetization I should itself produce.

Or, in other words: it should be possible to build up a reproduction of the magnetized piece of iron out of little magnets, the magnetic moment of each depending in a perfectly definite way on the force exerted on it by the other little magnets and by the external world, and all together producing the same effects in the external world as the piece of iron does.

In saying "it should be possible" I do not mean to imply that there is an obligation resting upon Nature to construct magnetizable objects in such a way that it is possible. One could not prove *a priori* that she does. One must take variously shaped pieces of magnetizable metal and observe their behavior in various impressed fields, and ascertain for each whether or not there is a function I . In so doing, one is liable to encounter very great mathematical difficulties. In fact, the difficulties are likely to prove insuperable unless the piece of metal is shaped in one or other of a few definite ways, and the impressed field is uniform and properly oriented.

Let us attack the problem from the other side, and enquire first whether it is possible so to shape a piece of iron and so to orient the impressed field, that the extra field due to the magnetization should vanish everywhere within the iron, and the actual field should everywhere be identical with the impressed field—so that although there is a function I differing from zero, yet $H_i = 0$ and $H = H_e$ everywhere inside the iron. This condition would be realized, if one could make an infinitely long straight rod and expose it to an infinitely extended uniform field parallel to its axis. It is very nearly realized along the middle of a wire several hundred times as long as it is thick, set parallel to the earth's field or along the axis of a solenoid somewhat longer than the wire itself. It is very nearly realized within the substance of a ring-shaped piece of metal pervaded everywhere by an impressed field following the curvature of the ring; a field of this character can be produced by wrapping a current-carrying wire around the ring.

In these cases, or rather in the ideal cases to which these are close approximations, the vectors H_e and I are uniform throughout the metal; the relation between their magnitudes is the relation between

"magnetizing field" and "intensity of magnetization," which is characteristic of the metal and is the cardinal fact of ferromagnetism.

Next we enquire whether it is possible so to shape the metal and so to orient the impressed field, that the actual field within the metal shall be uniform all through it even though not the same as the impressed field—so that I and H_e and H_i shall all three differ from zero, and the resultant H of H_e and H_i shall be uniform throughout the magnet. This condition is realized, if the piece of metal is an ellipsoid and the impressed field is uniform and directed parallel to one of its axes. In this case the ellipsoid is magnetized uniformly, and the extra field H_i which it produces within itself is uniform and oppositely directed,* "antiparallel," to the impressed field. The actual field H is uniform and points everywhere in the same direction as H_e , and its magnitude is equal to the difference between the magnitudes of H_e and H_i . The magnitude of H_i is proportional to that of I , as might be expected, so that

$$H = H_e - NI.$$

The factor N ("demagnetizing factor") depends upon the ratios between the axes of the ellipsoid, and Maxwell developed formulæ for it.

In these cases of ellipsoids, the relation between I and H_e , which is what the data usually supply, is not the true relation between the intensity of magnetization and the magnetizing field. However, the more significant relation between I and $H_e - NI$ can be deduced from the other by a simple graphical artifice. Ellipsoids of different shapes yield very different I -vs.- H_e curves; but the I -vs.- H curves into which these are translated in the aforesaid manner agree with one another, and with the curves obtained from closed rings or exceedingly long wires, very well indeed. Did they not agree, the whole theory would be upset; this procedure therefore is a manner of testing the theory.†

Incidentally, the field H_i produced by the magnet within itself may be far from insignificant. To take an example from Ewing:

* Unless the metal was not properly demagnetized before the application of the field.

† The artifice mentioned above consists in drawing upon the graph, on which orthogonal axes for I and for H_e have already been laid off, an additional axis passing through the origin and inclined to the I -axis at an angle of which the tangent is N . If now the I -vs.- H_e curve is plotted in the usual way, the value of H corresponding to any point P upon the curve is given by the length of the line drawn parallel to the H_e -axis and connecting P with the new axis.

In dealing with rods or other magnets shaped differently from ellipsoids, N may be determined empirically by plotting the I -vs.- H_e curve and drawing an axis so inclined to the I -axis that when the curve is referred to the new axis it coincides with the curve obtained with an ellipsoidal or ring-shaped magnet of the same material; the value of N is then the tangent of the angle between the new axis and the I -axis.

inside a sphere of soft iron exposed to the earth's magnetic field, H_i amounts to $84/85$ of H_e , so that only $1/85$ of the external field is active within the iron. Since the discovery of permalloy, this instance can be bettered. Within a sphere of suitably prepared permalloy exposed to a field of 10,000 gauss, 0.9996 of that field is counteracted by the magnetized volume-elements themselves.

This counterbalancing of part of the impressed field is sometimes called the *demagnetizing effect of the poles*—a rather unfortunate term, which affords me a pretext for discussing these alleged “poles.” The pole of a magnet is like the end of the rainbow; if one were to tunnel into a magnet to get the pole, one would not find it. Or, to draw a better simile from geometrical optics, the poles of a magnet are like virtual images behind a mirror. The virtual image is a point which we reach by retracing the light-rays backward to the surface of the mirror and then prolonging them straight ahead until they all intersect, even though the light-rays themselves actually came up to the mirror from some other direction; the magnet-pole is a point which we reach by prolonging the lines of force down into the substance of the magnet and carrying them on until they meet, although the lines of force actually supposed to prevail within the magnet may not converge at all. The poles, in fact, are like all the other entities supposed to exist inside a magnet—they are imagined, in order to describe and predict the field which the magnet produces outside of itself. For instance, the external field due to an ellipsoid magnetized parallel to an axis is precisely that which two “poles,” properly placed upon the axis and endowed with the proper equal amounts of positive and negative magnetism, would produce. If one chooses to visualize these “poles” rather than the ellipsoid, there is nothing to impede him.*

Again it is permissible, in the case of the ellipsoid and in some others, to visualize only the “magnetization of the surface”—to imagine the surface painted over with magnetism, laid on with a density governed by a certain law. At any point P on the surface of the ellipsoid, let \mathbf{I} represent the magnetization of the material, which as we have seen is a vector; let I stand for the magnitude of this vector; let ds stand for the area of a small element of the surface containing P ; let θ stand for the angle between the outward-pointing

* The inexactitude of this concept of “poles” leads to some curious lapses of logic in most expositions of the theory of magnetism (including, I am afraid, this one). Even in Maxwell we read: “The ends of a long thin magnet are commonly called its poles. . . . In all actual magnets the magnetization deviates from uniformity, so that no single points can be taken as the poles. Coulomb, however, by using long thin rods magnetized with care, succeeded in establishing the law of force between two like magnetic poles.”(!)

Some use the terms “poles” or “polestrength” in the sense assigned to the word “magnetism” on p. 298.

normal to ds and the vector \mathbf{I} . Magnetism in the amount $I \cdot ds \cdot \cos \theta$ is to be spread upon ds ; magnetism is to be spread over the surface of the ellipsoid with surface-density $I \cdot \cos \theta$. This film of "magnetism" would produce, everywhere outside of the ellipsoid, the same field as the poles or the continuous magnetization which we have imagined as existing inside the ellipsoid. Furthermore, it has a firmer basis in experience than do the poles. For, if a beam of polarized light is directed against the surface of a magnetized ellipsoid, the reflected beam is curiously altered; this effect, known by the name of its discoverer Kerr, is sometimes extremely complicated, but in magnitude it is always proportional to the value of the imagined quantity $I \cdot \cos \theta$ at the point where the reflection occurs; and by promenading a spot of light over a magnetized piece of iron and analyzing at every point the reflected beam, one can actually find how $I \cdot \cos \theta$ varies all over the surface. This property endows the vector I with a physical reality.

There is still one of the effects which a magnet produces outside of itself, which requires our attention; did it not exist, magnets would not play nearly so great a rôle as they do in the life of the world.

Hitherto I have implied that one maps out the external field of a magnet by exploring it with some one of the known field-measuring devices, of which there are several: the magnetometer needle, the bismuth wire which changes its resistance according to the field impressed upon it, the plate of glass which rotates a traversing beam of plane-polarized light to an extent proportional to the field. There is another method essentially different from these, and capable of measuring something which they cannot. One may set up a loop of wire in the neighborhood of the to-be-magnetized piece of metal; suddenly impress the magnetizing field; and measure the sudden rush of charge around the loop. This rush of charge is proportional to the mean value of the magnetic field thus suddenly created in the region enclosed by the loop.* One might map a field in this manner; but that is not the unique feature of the method.

We consider a special and actual case. Take an unmagnetized ring of iron; cut out a thin segment, leaving two nearly parallel end-surfaces facing one another across a narrow gap; take a loop slightly larger than the cross-section of the ring, suspend it in the gap,

* The E.M.F. around the loop at any instant is equal to the time-derivative of the surface-integral, over any surface bounded by the loop, of the component of the magnetic field normal to the surface; in technical language it is equal to the rate of change of the flux of magnetic field through the loop. The rush of charge is equal to the quotient of the time-integral of this E.M.F., which is the difference between the initial and final values of the surface-integral, by the resistance of the loop.

parallel to the end-surfaces; apply an impressed field H_e by sending a current through a coil wrapped around the ring. The rush of charge in the loop testifies that the field established in the gap is vastly greater than H_e , a fact which can be confirmed by the magnetometer or any other field-measuring device. The field in the gap is, in fact, the resultant of H_e and a field due to the magnetized iron. We call it B . Replace the segment, closing the ring; encircle the restored segment with the loop as with a collar; repeat the experiment (after carefully demagnetizing the ring, so as to start afresh from the same condition). The rush of charge is the same. The apparent inference is, that the field B continues to subsist inside the iron forming the closed ring; and the method of the loop seems to be competent to measure, if not the actual force within the metal, at least the average of its values—which would contradict in part my former statement that the field within the iron is unreachable by measurement.

The contradiction involves one of the most confusing assumptions in the theory of ferromagnetism. The field B is greater than the impressed field H_e , whereas the actual field H , which we have been postulating within the iron in order to explain its magnetization, is smaller than H_e . To prove this for the ring might be difficult, since it is a property of the complete ring that the field due to its own magnetization is zero everywhere outside of it as well as inside (so that, incidentally, the method of the loop is the only one giving even an intimation that the ring is a magnet). With an ellipsoid the demonstration is easy. Wrap the loop like a girdle around the middle of an ellipsoid of iron, and suddenly magnetize the iron by impressing a uniform field H_e parallel to one of its axes and normal to the plane of the loop. Measure the rush of charge; it attests that the field established through the loop is much greater than H_e . But the field within the iron, as we have seen already, has been set equal to $H_e - NI$, hence to a value smaller than H_e , in order to account for the field outside.

It is necessary, therefore, to add a third vector B to the pair of vectors I and H which we have already conceived as existing in the depths of the magnet. It is this vector, the alteration of which governs the rush of charge which occurs through a loop encircling the magnet when the magnetization is changed. The rush of charge is proportional to the change in the mean value of B throughout the magnet in the plane of the loop—not to the mean value of H . Making this the definition of B , and considering all the data assembled from experiments on rings and ellipsoids and rods of various proportions, it is found that the observations made upon their external fields by

field-measuring devices and the observations made by the method of the loop are all reconcilable with one another, provided that the vector B is made parallel to I and H and equal to

$$B = H + 4\pi I.$$

The vector B is known as the *induction*. The relation between B and H is often plotted instead of the relation between I and H ; naturally if either relation is known the other can readily be found. The ratio of the magnitudes of B and H is called *permeability* and denoted by μ ; the ratio of the magnitudes of I and H is called *susceptibility* and denoted by κ or σ or χ .

One might think that this quantity B should be identified with the magnetic field which is supposed to exist within the metal and to magnetize it. Though all the textbooks beseech the student not to confuse the induction with the field (he is usually asked to imagine himself digging variously shaped infinitely small holes within a magnet, and putting an instrument into each to measure the magnetic force inside it), the distinction has an obstinate way of not becoming clear. We should get just as self-consistent sets of curves if we were to plot I against $(H_e + H_i + 4\pi I)$ as we do when plotting I against $(H_e + H_i)$; it would merely be tantamount to adding 4π to the "demagnetizing factor." As a matter of fact nearly everyone, as soon as he begins to theorize about the state of affairs inside magnetized bodies (or polarized dielectrics), promptly assumes that the acting field is something different from the resultant of H_e and H_i . Some make it equal to $(H + \frac{4}{3}\pi I)$, attributing the term $\frac{4}{3}\pi I$ to an action of the molecules which are neither very close to nor very far from the point where the field is being evaluated. Some (Weiss and his many followers) make it equal in ferromagnetic metals to the sum of H and a term nI , the factor n being so enormous that the postulated field is millions of times as great as H and thousands of times as great as B . The extra field, they say, is "not magnetic"; but this distinction is more obscure than the other. Nobody really knows what the field inside a magnetized solid is. The best policy is to continue plotting I and B as functions of H , regarding H as the independent variable sanctioned by tradition.

B. THE RELATION BETWEEN INTENSITY OF MAGNETIZATION AND MAGNETIC FIELD

Since all of the actions of magnets are interpreted by supposing that in every magnetizable substance the intensity of magnetization is controlled by the magnetic field in a definite and peculiar way—

that for every magnetizable substance there is a distinctive I -vs.- H relation—it is evident that this relation, if it exists, must be the fundamental fact of magnetism. The first object of research in ferromagnetism is to discover it for all of the ferromagnetic materials; the second, to devise for each of these materials a model, accounting for the particular form of I -vs.- H relation which it displays.

On setting about to collate the recorded samples of I -vs.- H curves, one promptly encounters the last and greatest of the troubles of ferromagnetism. There are infinitely many such curves to be collected, for there is a limitless variety of ferromagnetic substances!

This is not always realized, because of the unfortunate practice of referring to "the three ferromagnetic metals, iron, nickel and cobalt," as though there were but three I -vs.- H relations to be determined. But in addition, there are ferromagnetic alloys: binary alloys of iron with nickel, of nickel with cobalt, of cobalt with iron; ternary and yet more complex alloys comprising these and other elements, or consisting entirely of elements none of which by itself is ferromagnetic. Anyone acquainted with the diversities of alloys would be prepared to find a truly vast variety of qualities exhibited by these; and he would not be disappointed. Indeed, an alloy may contain one of its elements in so small a proportion as to appear quite negligible—so small, as to be considered a mere casual impurity—so slight, as to be difficult to detect and difficult to expel—and yet so great, as to influence the magnetization in the most drastic fashion. Iron containing a fraction of a per cent of carbon differs as much from pure iron, in regard to its magnetic properties, as either differs from nickel. (Perhaps even what is now called "pure" iron contains a minimal amount of some undetected yet potent impurity, the ultimate removal of which will reveal a whole new set of phenomena!) So there is not a triad, but a multitude of ferromagnetic substances, each of which may be expected to have a distinctive I -vs.- H relation of its own.

But for each of these substances there is, as it turns out, not one but a legion of I -vs.- H relations. The curve depends very much on the temperature of the sample—to such an extent, indeed, that as the temperature is raised, the ferromagnetism varies rapidly, diminishes, and finally vanishes. The curve depends also upon the mechanical stresses prevailing in the material, compression and tension and torsion and the complicated combinations of these. It is also liable to be altered by an electric current flowing in the material.

Degree of crystallization likewise matters a great deal. Most of the samples of metal used in the past have consisted of very great numbers of very small crystals, millions of them to a cubic centimeter.

Recently it became possible to make individual crystals so large that one of them, or an aggregate of a few, is by itself large enough to serve as a sample for magnetic testing. The I -vs.- H relation for an individual crystal is very different from the relation for a mass of tiny crystals of the same material. In fact, the vector \mathbf{I} is usually not parallel to the vector \mathbf{H} . A magnetic field, applied to an ellipsoid cut from a single crystal, magnetizes it askew unless the field, and an axis of the ellipsoid, and an axis of the crystal happen to be all parallel to one another. In the polycrystalline mass, these deviations between direction of field and direction of magnetization must be averaged, and cancel one another out; for otherwise, the universally made assumption that I is parallel to H would not have been effectual. No doubt it is fortunate that Nature, with a rare benevolence, simplified the data first presented to the students of magnetism by this averaging and this cancellation. We cannot however conclude with safety that an assemblage of small crystals will behave just like an assemblage of equally many large ones. Evidently the size of the crystal must influence its I -vs.- H relations, or else the boundaries between adjacent crystals affect the magnetization, or there is something inherent which changes concurrently with the degree of crystallization. At any rate, whenever the crystallization of a sample is varied, the I -vs.- H curve is liable to feel it.

Composition and strain, temperature and current, state of crystallization—one must be prepared to find a new way of dependence of I upon H for each combination and every gradation of these; and yet the half has not been told. Whenever stress or heat are applied to a magnetizable substance, they alter its I -vs.- H relation, *not merely while they are being applied, but after they are withdrawn*. After such an experiment one may restore the original temperature and the original stress or freedom from stress, but the material is no longer quite the same. Vibrations and concussions, compressions and tensions and twistings, bending and tapping and cold-rolling and hammering, heating and cooling, annealing and quenching, *the very act of magnetization itself*—each of these is liable not merely to affect the I -vs.- H curve while it prevails, but to transform the substance permanently into another and a distinct ferromagnetic substance, with a system of magnetization-curves distinct from what the sample showed beforehand.

If we could see into the penetralia of a piece of iron, and discern the conditions and the arrangements of its atoms, it is probable that we should see that every such agency leaves behind it some definite and enduring change; and then we should not wonder at (for example)

the fact that an iron wire, which undergoes the experience of being violently pulled and then relaxed, displays very different I -vs.- H curves before and after this adventure. In certain cases we do observe some sort of an attendant change, as for instance when the iron wire has been stretched so forcefully that it is permanently lengthened, or cold-rolled so vigorously that the X-ray diffraction-pattern due to its little crystals is affected. In other cases we observe no concurrent change whatever, and are forced to assume that there has been an internal alteration of the metal, for which there is no evidence beyond the testimony of the changed magnetization-curve. In the same way, we are prone to assume that when "the burnt child dreads the fire," something is altered within his brain-cells, for which there is no evidence except his change of conduct. As a rule, one would not speak of the brain-cells; one would say that the child has a memory of the painful burn. The ferromagnetic substance also changes its conduct after each experience, as though it remembered. No one, I presume, supposes that it actually has a consciousness which remembers; but the actual responsible alteration, whatever it may be, is often as far beyond detection as the alteration in the brain-cells. The resulting change in conduct, the result of this "memory" of the metal, is what is known as *hysteresis*.

All this makes the designing of a model for a ferromagnetic substance a very difficult and perplexing problem indeed, as we shall discover in due time. For the moment we are concerned only with knowing how much of the biography of a piece of metal must be recorded, in order to give background and value to a determination of its I -vs.- H curve. A curve inscribed "*This is the I -vs.- H curve for iron*" would not be worth much, no matter how carefully it had been determined nor how nearly pure the iron had been. At this point the physicist must betake himself to the foundry and the rolling-mill, and confer with the metallurgist, and learn the usage of a number of uncouth words such as *swaging* and *sintering* and *cold-working* and *quenching*, and grasp the distinction between cast-iron and wrought-iron and pig-iron and soft steel and hard steel, and observe a number of processes which were discovered so long ago that originally they were practiced without the least assistance from the guiding hand of pure science. The curve for his sample of metal must be labelled with the processes which the sample underwent before and after it came into his hands. Even yet it is not completely settled how many of the details of these processes should be recorded, nor how far back the history of the sample should be traced. One piece of knowledge, however, dispenses us from the risks of this uncertainty; it is known that a long-continued

annealing,* followed by a gradual cooling, obliterates the traces of earlier experiences; and consequently a sample of unknown (or known) antecedents can be restored, by putting it through this process, to a standardized initial state.

Imagine, then, a sample of nickel which since its latest rejuvenation by annealing has undergone a recorded set of experiences; for instance, that it has been "stretched almost to the point of rupture, bent into a circle, and allowed to restraighten itself" (I quote an actual case investigated by R. Forrer). It might now be thought that, so long as the greatest care is exercised to avoid subjecting the metal to new stresses, concussions or heatings, the I -vs.- H curve would be fixed for good. Not so! for in order to determine the I -vs.- H curve, the metal must be magnetized; and magnetization, like stress and heating, is one of the events that leave an imprint, one of the experiences which the metal remembers. If two I -vs.- H curves are measured in succession, the second is generally not the same as the first; during the process of ascertaining the first, the material was changed into a new one. To predict or classify an I -vs.- H curve, one must not only know the composition of the substance, not only have records of its entire mechanical and thermal history since it was last rendered forgetful of its past by annealing, but also have the protocol of all its magnetizings since the last occasion when it was "completely demagnetized"—whether by the annealing which effaced all the memories, or by the gentler process † prescribed by Ewing which cancels the imprints of past magnetizations without destroying those of past stresses and heatings.

To make some choice among this staggering mass of data, it is suitable to concentrate one's attention on two, or rather on one and a group, of the infinite multitude of curves. The first of the chosen curves is obtained by applying to a sample which is freshly demagnetized a magnetizing field H , which is increased by consecutive small steps, and measuring the field of the magnet after, or (by the method of the loop) the increase of the induction in the magnet during, each of these steps. From either of these sets of data, after making the allowances and the reductions indicated in the first section of this article, one may determine the I -vs.- H curve for steadily-increasing magnetizing fields applied to a piece of metal initially demagnetized.‡

* I use the word "anneal" to denote a long-continued maintenance at a high temperature, irrespective of the rate of cooling thereafter.

† By applying an alternating magnetic field of which the amplitude is at first greater than any field which has been applied to the magnet, and thence diminishes gradually to zero. However, the effect of this process is not quite thoroughgoing.

‡ There is a risk that the increase in magnetization at a certain step may be so great that, when due allowance is made for the demagnetizing effect of the magnet upon itself, it will be found that H has actually decreased in spite of the increase of H .

This is what is sometimes called the normal magnetization curve, sometimes the initial curve; I will adopt the latter term.

At this point it is well to recall that most of the curves actually found in the literature are B -vs.- H curves, not I -vs.- H . Since in the right-hand member of the equation $B = H + 4\pi I$ the second term is usually enormously greater than the first, a B -vs.- H curve usually looks exactly like an I -vs.- H curve plotted on a smaller scale. At very high fieldstrengths, however, a B -vs.- H curve continues climbing upward with a constant slope while the corresponding I -vs.- H curve runs parallel to the H -axis.

The Initial Curve

The form of the initial curve is peculiar and distinctive. Departing from the origin of the (I , H) coordinate-plane, it ascends, bends upward, passes through a point of inflection, bends over but never quite turns downward; it goes off towards a horizontal asymptote, toward a maximum or *saturation* value of magnetization. Nearly all initial curves display these features, the point of inflection and the saturation; but in all other details, in the lengths and curvatures of the arcs before and after the point of inflection, in the scale of the curve and of its parts, they differ very much from one substance to another, and are altered very much by mechanical and thermal treatments.

Well-annealed substances, iron and nickel and permalloy for instance, display curves which tempt the onlooker to divide them into three segments: a slowly-rising and eventually upward-bending arc starting from the origin, a relatively steep-climbing portion including the point of inflection, a final arc drawing itself close up to the asymptote. A good example is shown in Figure 1. The distinction is accentuated by the hysteresis-loops which originate from the various points of the curve. In the prevalent theories of magnetization, as we shall eventually find, these segments are supposed to result from different processes occurring inside the metal. I will therefore adopt this separation of the curve into three parts, warning the reader to remember that at best there is always something arbitrary in subdividing a continuous curve, and at worst there are substances in which the division into three segments becomes quite impossible to make.

The first segment, extending from the origin to what some call the instep of the curve, may be regarded as a parabolic arc so long as the field is rather low—for iron and nickel, inferior to about one gauss; and for these metals it is sensibly a straight line so long as the field is below say a tenth, or to be safe a hundredth of a gauss.

This rather nebulous statement might be made precise by expressing I as a power-series in H , after this fashion:

$$I = aH + bH^2 + cH^3 + \dots,$$

and citing experimental values of the coefficients a, b, c, \dots ; from this a student equipped with a measuring-instrument could determine

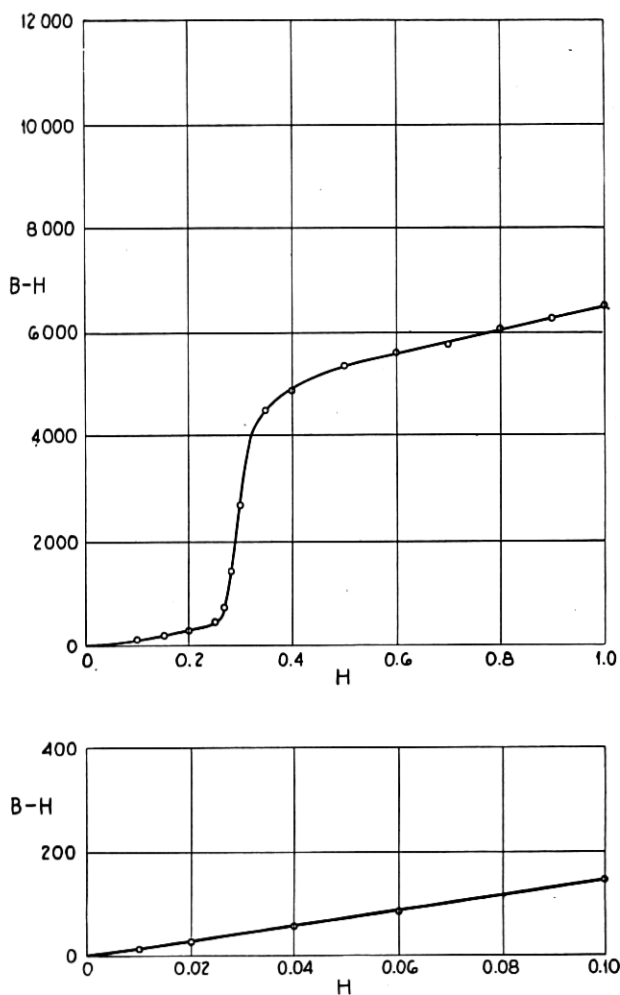


Fig. 1—Initial curve for a permalloy, displaying the three segments. The lower curve is part of the first segment of the upper, on a larger scale. (After L. W. McKeehan.)

the value of H at which the terms after the first become so small that with his apparatus he could not detect them. I mention this to

show how valueless is the mere statement that "the early portion of the magnetization-curve is very nearly straight."

Values of the coefficient a , which is frequently called *initial susceptibility*—the corresponding value of $(1 + 4\pi a)$ is called *initial permeability* and denoted by μ_0 —are rather often determined; it is an important constant of each material. Some pairs of values of a and b are quoted by Ewing, others by Bidwell, and some others were determined by the pupils of Weiss. According to one of these latter (Renger) the values for very pure freshly-annealed iron at room-temperature are: $a = 49.9$, $b = 108$. Tempered steel however yielded values of 2.23 for a , and 0.032 for b ; from which, and from a mass of other observations on metals hardened by stretching, one sees that the effect of hardening is to lower a a great deal and b a great deal more, so that the curve slopes less sharply upward and does not begin to bend appreciably for a much longer way. I cannot quote all of the relevant data; but it is worth remembering that Rayleigh made measurements so delicate that he was able to follow the curve (for unannealed iron) all the way from .04 to .00004 gauss. Over this range his magnetometer reported no variation in the ratio of I to H . For nickel the detectable upward curvature commences at a much higher fieldstrength—five gauss, according to Ewing.

The alloys of iron and nickel, containing more than 30 per cent of the latter element, develop extraordinary magnetic properties when they are submitted to certain heat-treatments,* as G. W. Elmen discovered towards 1915. For these, the first segment of the magnetization-curve shrinks to a small fraction of the length it has for iron; the two-term formula

$$I = aH + bH^2$$

becomes visibly inadequate at 0.02 gauss, as the curve sweeps upward into its rapidly-rising stage. The value of a for some of these "permalloys" is as great as 8000, the value of b as great as 4000.

As the value of H is increased the later terms in the power-series for I bulk larger, and eventually the first segment of the curve passes over into what I have called the second. In this second section the ratio of I to H rapidly rises, and attains the enormous values which form one of the distinguishing marks of ferromagnetic substances, and are responsible for much of their utility in the world of engineering. Plotted against H , the ratio of I to H appears as a curve with a high

* For samples of a certain specified shape and size, this is the heat treatment which was recommended: "They are first heated at about 900° C. for an hour and allowed to cool slowly, being protected from oxidation throughout these processes. They are then reheated to 600° C., quickly removed from the furnace, and laid upon a copper plate which is at room-temperature."

sharp maximum, and so also does the more-commonly-plotted ratio of B to H , the permeability μ .

$$\mu = B/H = (H + 4\pi I)/H = 1 + 4\pi(I/H).$$

Pure iron attains much higher values of permeability than does either of the other metals which can be ferromagnetic when pure. By careful purifying and long annealing, T. D. Yensen elevated μ_{\max} .

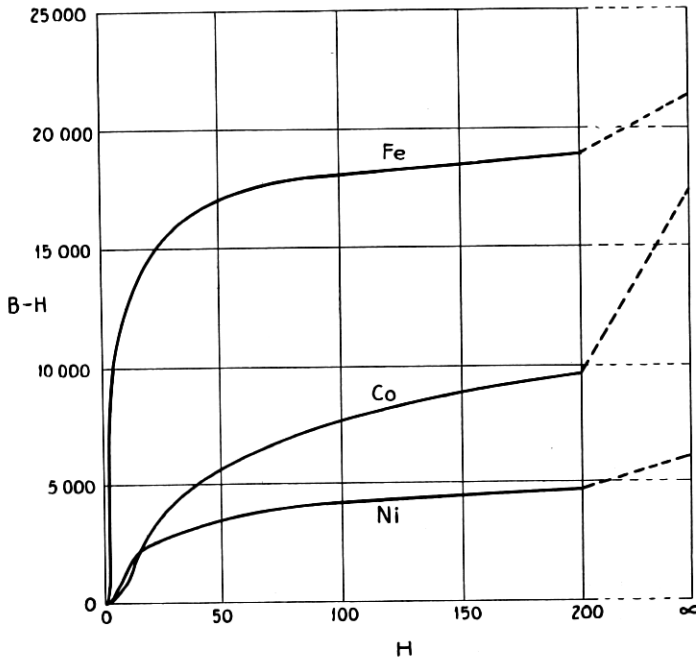


Fig. 2—Initial curves for annealed iron, nickel, and cobalt.
(After L. W. McKeehan.)

for iron to 19000; the best recorded values for nickel and for cobalt are very considerably lower. Certain alloys of iron, however, leave the pure metal far in the rear; by slight admixtures of silicon (between 0.15 per cent and 4 per cent) Yensen produced materials which, after being melted in vacuo, annealed at a high temperature and very slowly cooled, developed a value of μ_{\max} as high as 66500. These in their turn were surpassed by the permalloys of G. W. Elmen for which μ_{\max} ascended past 100000.

Other alloys of iron, and in fact nearly all of them, are much inferior to the pure metal in respect of μ_{\max} . Carbon in particular is

pernicious; one per cent of this element mixed with iron brings the maximum permeability down to 350. A few per cent of manganese mixed into iron reduces I/H to the nearly-constant value of .03. Tempering, cold-working, forging, and drawing all tend to reduce the permeability. Since these processes render the metal harder in the literal sense of the word, the change which they imprint upon the magnetization-curve is called by association of ideas a "magnetic hardening." As μ_{\max} is reduced by any of these processes, the contrast between the three segments of the I -vs.- H curve diminishes, and in some cases there is scarcely more than the point of inflection left to mark the passage from the initial to the final range of the curve.

The final approach to saturation conforms to the law

$$I = I_{\max} \cdot \left(1 - \frac{c}{H}\right).$$

The value of the constant c is large for magnetically-hard materials, and small for the well-annealed samples for which the tripartite division of the I -vs.- H curve is obvious. In iron (I quote Weiss) saturation is approached within one promille at a field strength of 5500, in nickel at $H = 10000$. In permalloy it must be approached as closely with a field of a few dozens of gauss.

The saturation-intensity of magnetization, or *saturation* for short, is much more nearly independent of the present hardness and the past mechanical and thermal treatments of the material than the other features of the initial curve—much more nearly, therefore, a function of the chemical composition exclusively, than is any other single nameable magnetic quality. For this reason it is possible to present such a Table as the accompanying one with comparatively few qualifications. The first column of figures contains values of I_{\max} obtained near room-temperature; the second, values measured at the temperature of boiling hydrogen, inserted here for future reference.*

* These values may be described as the "saturation magnetization of a cubic centimetre" of the materials in question. Dividing each by the density ρ of the material, we obtain the "saturation magnetization per gramme." Multiplying this by A , the atomic weight of the element or molecular weight of the compound (if the material is of either sort) we get the "saturation magnetization per gramme-molecule." This last is the quantity most often tabulated, being sometimes expressed in "magnetons" (units equal to 1126 C. G. S. units; cf. page 353). It may be advisable to recall that an *isolated* cube containing one cubic centimetre, or one gramme, or one gramme-molecule of material would not acquire the magnetization in question at any finite field, since it could not be magnetized uniformly.

TABLE

	$I_{\max.}$ (20° C.)	$I_{\max.}$ (20° K.)
Iron.....	1706	1742
Nickel.....	479	505
Cobalt.....	1412	
Alloy Fe ₂ Co.....	1880	
Permalloy Ni 78.5 per cent, Fe 21.5 per cent.....	870	
Heusler alloy Cu 75 per cent, Mn 14 per cent, Al 10 per cent.....	222	
Magnetite.....	490	518
Pyrrhotine.....	62	

The dependence of the initial curve upon temperature and strain is great and important; but it is expedient to reserve discussion of these variations to later sections.

The Hysteresis-Loops

Any ferromagnetic material has an infinite variety of hysteresis-loops, almost any one of which may turn up in practice; but I will limit this discussion to those obtained by a particular procedure, thus: Commence by demagnetizing the sample—increase H gradually to any desired value, denote this by H_0 —decrease H gradually to and through zero, reversing it and bringing it to the equal and oppositely-directed value ($-H_0$)—return gradually to $+H_0$ —return to ($-H_0$)—and so over and over again, ten or twenty times at the least. The point representing I as function of H , or B as function of H , traces out at first an arc of the initial curve extending as far as H_0 ; thenceforward it travels in sweeping detours passing around and around the origin, successive ones becoming more and more closely alike, until at last it settles into a routine of tracing the same oddly-shaped loop over and over again. I have spoken of the “memory” of the magnetic material; this process recalls the consolidation of memory into habit. The final habitual loop thus attained is the particular and chief *hysteresis-loop associated with H_0* . Demagnetizing the sample afresh and repeating the process with a new value of H_0 , one gets another loop*; and in this way a family of hysteresis-loops can be determined, one for every point along the initial curve.

So long as H_0 is so low that the initial curve does not depart appreciably from a straight line, the hysteresis also is inappreciable; the point representing I (or B) as function of H goes back and forth

* The demagnetization may be dispensed with, if the new value of H_0 is greater than the prior one.

through the origin along the line of slope a (or μ_0). For this reason, the sensibly-linear part of the initial curve is often called the "reversible part." When it passes over into the perceptibly-upward-turning part, the hysteresis-loop becomes perceptible. Over a certain range its area varies as the cube of H_0 , and Weiss gives this formula, in which the coefficient b is used with the same meaning as heretofore:

$$\text{Area of hysteresis-loop} = \int H dI = \frac{3}{4} b H_0^3.$$

In the second segment of the initial curve, the loop swells out to its fullest amplitude. This forms one of the reasons for the division of that curve into three parts; the middle one is sometimes called the "irreversible portion" of the curve. There is no formula available in this region, except the oddly though not universally effective one discovered by Steinmetz, in which the area of the loop is related not to H_0 but to the maximum value B_0 attained in the cycle. This "law of Steinmetz" reads *

$$\text{area of loop} = \eta B_0^{1.6}.$$

Values of the constant η are frequently quoted in describing magnetic materials.

When H_0 is carried far into the third stage of the initial curve, so that in each cycle I approaches within a few per cent of $I_{\max.}$, the hysteresis-curve assumes the form of a wide loop prolonged at its northeast and southwest corners (I use the analogy of a map) by long slender projections which narrow down into mere lines. So long as I is nearly equal to $I_{\max.}$, the point tracing the I -vs.- H curve passes back and forth along nearly the same path. The final stage of the initial curve is therefore also called "reversible." The Steinmetz formula here becomes invalid.

The reason for laying so much stress on the areas is well known. When a piece of magnetizable metal is carried through a cycle of magnetization, for instance by varying the current through an encircling solenoid in a cyclic manner, the battery supplying the current is found to have expended an amount of energy $\int H dI$ per unit volume; and the metal is found to be warmed to a degree indicating that an equal amount of heat energy has appeared within it.

* The formula of Steinmetz is more general; it applies to hysteresis-loops executed between any two (not overly great) values of induction B_1 and B_2 , and for these assumes the form

$$\text{area of loop} = \eta \left(\frac{B_1 - B_2}{2} \right)^{1.6}.$$

It is clear that B_1 and B_2 must be given opposite signs if directed in opposite senses.

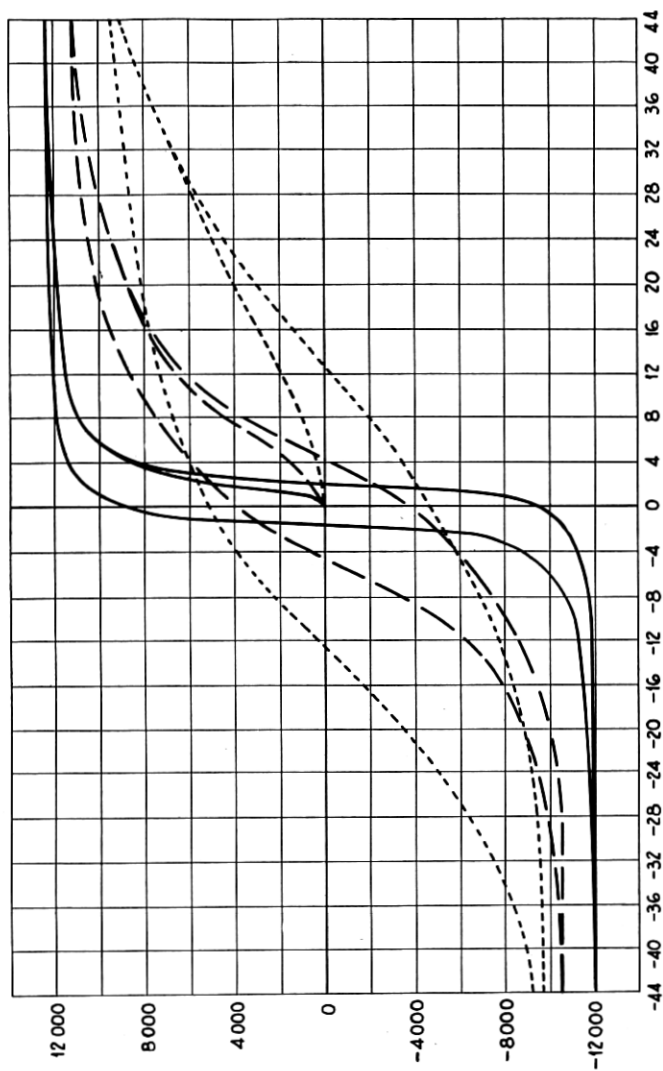


Fig. 3—Initial curves and hysteresis loops for annealed iron (continuous), harddrawn iron (dotted), and hard steel (dashed). (After F. Auerbach.)

We do not know how the transformation from electrical energy to thermal energy was effected; but we do know that so much electrical energy vanishes, and that so much thermal energy appears. A broad hysteresis-loop therefore signifies that there will be much dissipation of energy if the sample is exposed to cyclic magnetizing forces, as usually happens in electric machinery in which magnets play an important rôle; and the heat developed is not merely a sign of energy gone to waste, it is often detrimental to the material, and a bad contribution to that unforgettable history which the magnet is forever piling up. For these reasons the discovery of a new ferromagnetic material of low hysteresis is always welcome.

As a rule, narrow hysteresis-loops go with high values of initial permeability, and with initial curves easily divided into three stages, and with early saturation; and wide loops go with initial curves which rise slowly and bend upward slowly and display no sharply-marked second segment and approach very tardily to the saturation value. Magnetic hardening, in the sense which I earlier defined, accentuates hysteresis; and the agencies which bring it about—tempering and mechanical hardening and the admixture of certain elements in small quantities, such as carbon into iron—widen the loops and augment the generation of heat. These effects are frequently described by giving measurements of the heat W generated in a single cycle of magnetization in which B is carried back and forth between standard values $+B_0$ and $-B_0$ of the induction-measurements which in their turn are cited by giving the corresponding values of $W/B_0^{1.6}$, the quantity η of the formula of Steinmetz.* The value of this quantity is only .00032 for very pure well-annealed iron, leaps to 0.015 when one per cent of carbon is added, leaps again to 0.034 when the so-constituted steel is tempered; while the addition of silicon to iron, the very process which raises $\mu_{\max.}$ to values excelled only by permalloys, brings the value of η down to .00011. The permalloys themselves are still more eminent in this regard, some of them having hysteresis-loops only a sixteenth as great in area as those of pure annealed iron.

To give the area of a loop is not always sufficient; its shape and orientation are very important for theory and for practice. The agencies which harden a material not only widen its hysteresis-loops, but rotate them clockwise around the origin, as the figures show. This rotation tends to decrease the intercept of each loop upon the

* I should emphasize that for many materials the "law of Steinmetz" is not accurate, so that strictly one should plot the actual curve of hysteresis-loss-*vs.* B_0 , instead of making a single measurement and using it to determine η by the assumption that the "law" is valid.

axis of I or B , and increase the intercept upon the axis of H .^{*} The former of these intercepts, representing as it does the magnetization which the metal retains when the external field has been reduced to zero, is known under the names of *residual magnetization* and *remanence* and *retentiveness*. The last two of these words, and also *residual magnetism*, are used in a general sense, to denote the property of not losing magnetization altogether when the magnetizing field is withdrawn. The intercept on the axis of H , representing as it does the force which must be applied oppositely to the direction of the prior magnetization in order to annul it altogether, is known under the names of *coercive force* and *coercivity*.

Residual magnetism was the first of magnetic phenomena to come under human notice. If the pieces of magnetite (lodestone) in the fields of Asia Minor had not been able to retain the magnetization which they had acquired in past ages, the Greeks would never have observed nor produced a magnetized metal; if steel needles rubbed against pieces of magnetite or held parallel to the earth's field and "smartly tapped," as the English textbooks say, could not retain the magnetic moment they so acquire, there would have been no compass-needles; the discovery of magnetism would probably have waited upon that of electric currents. Residual magnetism is the property to which the intercept of the hysteresis-loops gives a definite and definable meaning.

The greatest remanence, usually called *retentivity*, is attained after the material is magnetized to saturation. It may be as much as three-quarters of $I_{\max.}$, or more.[†] Occasionally one finds samples of materials for which the ratio of the greatest remanence to $I_{\max.}$ lies close to some simple fraction—in nickel, for instance, to $1/2$. The greatest *ratio* of remanence to previously-attained magnetization, however, is obtained by choosing H_0 somewhere in the second segment of the pristine curve. In fact, there may be a long range of the second segment over which the difference ($I_1 - I_2$) between the ordinates of the initial curve for any values H_1 and H_2 of the magnetizing field is practically equal to the difference between the values of the remanence in the hysteresis-loops for which $H_3 = H_1$ and $H_0 = H_2$ respectively. In other words: along the second segment of the curve, whatever added magnetization is given to the metal by increasing

^{*} This is not a universal rule; samples of electrolytic iron studied by E. Gumlich and W. Steinhaus (*E. T. Z.*, 36, pp. 675-677, 691-694; 1915) which had been annealed at constant temperatures and cooled at various rates displayed intercepts on the I -axis which were much lower when the cooling had been rapid than when it had been slow; but the intercepts on the H -axis remained nearly unchanged.

[†] Ewing records an instance of remanence 0.96 as great as prior magnetization (hardened nickel under strong compression).

the field is *kept* almost intact when the field is annulled. Near the beginning and near the end of the curve, the magnetization which is conferred upon the metal by the field departs with the field. Ewing's theory of magnetization is strengthened by this fact.

The greatest remanence, as I have intimated, occurs with magnetically-soft materials. Magnetic hardening tends to augment the coercive force at the expense of the remanence. It does not follow that in constructing a good strong permanent magnet one should take a piece of well-annealed iron or permalloy. A substance of low coercive force is liable to lose its magnetization not only when it is exposed to a weak counteracting field, but also when it is bumped or jarred. A magnet which ceases to be one when dropped on the ground is not of much use in the compass or the automobile.* Great coercive force is much sought after in designing permanent magnets, and the alloys developed for this purpose are at the opposite pole of the ferromagnetic world from the permalloys. The maximum coercive force is attained after the material is magnetized to saturation; for it the name *coercivity* is used and should be reserved. The coercivity of iron, which when the metal is very pure and well annealed may be as low as 0.5 gauss, is elevated past 50 by alloying with one per cent of carbon, past 60 by a few per cent of tungsten, past 80 by a few per cent of molybdenum, up to 370 by amalgamating the iron with mercury. For the permalloys the values drop below 0.05. These figures naturally relate to samples already magnetized to saturation.

Other I-vs.-H curves

Still other *I-vs.-H* curves are obtained in special ways, a few of which I will mention.

If during the measuring of an initial curve the sample is continually shaken, or if after each change in magnetizing field it is traversed by a damped alternating current before the magnetization is read, the *I-vs.-H* curve rises very swiftly from the origin; it seems as if the first segment had been suppressed, the second rendered steeper than for the undisturbed sample. Fantastically high values of the ratio I/H are sometimes obtained in this way. Such curves are sometimes called "ideal curves," owing to an impression that they represent the true law of magnetization undisguised by accidental (?) influences.

If in the process of measuring a hysteresis-loop the observer stops

* There is an additional reason for not making permanent magnets out of substances of low coercivity. Suppose an ellipsoid of such a substance magnetized to saturation by an external field H_e ; let H_e be reduced gradually to zero; the field $H = H_e - H_i = H_e - NI$ passes through zero long before H_e does, and when H_e finally falls to zero the value of I has fallen far below the true remanence unless the *I-vs.-H* curve runs nearly parallel to the axis of H .

short after the first reduction in magnetizing field following the attainment of the maximum applied field (the one which I above called H_0), returns to H_0 , and alternates the field several times between H_0 and the inferior value $H_0 - \Delta H$, he finds that the magnetization settles down to a routine of alternating between definite values I_0 and $I_0 - \Delta I$. The limiting value of the quotient $\Delta I / \Delta H$, for small values of ΔH , is known as *reversible susceptibility*. It is a function of I ; that is to say, if we select any particular value of I we always get one and the same value of $\Delta I / \Delta H$ when we impart that value of I to the metal, whether by mounting to it along the initial curve or the "ideal" curve or coming to it along any hysteresis-loop.

If the hysteresis-loop is described very rapidly and continuously, it retains its shape surprisingly closely until the frequency is raised into the hundreds of thousands. The initial permeability is still more nearly unaffected by rapidity of variation of field, remaining sensibly unchanged until the range of radiofrequencies is reached and passed. In the range of light-frequencies, however, it is reduced to unity.*

Magnetostriction

"Magnetostriction" is the clumsy name given to the divers very inconspicuous strains in a magnetizable body, brought about by the process of magnetizing it. As they are exceedingly small—a variation of any linear dimension amounting to four parts in one hundred thousand would be ranked as a remarkably big one—and as magnetizable materials are usually investigated in the form of long thin rods, the change in the length of such a rod resulting from a magnetic field applied parallel to its axis ("Joule effect") is the only magnetostrictive change which is often mentioned. Changes in the dimension normal to the field do, however, take place; a rod which expands lengthwise in a longitudinal field will contract sidewise, and *vice versa*. It used to be thought that the change in length just compensates the change in thickness, so that the net change in volume would turn out to be nil; but this turned out too simple to be true. A wire exposed to a longitudinal field and traversed by an electric current will twist itself (the "Wiedemann effect"). This occurs because the impressed field and the circular field due to the current itself are compounded with one another into a resultant pointing slantwise to the axis, so that any particular "line of force" can be visualized as winding in a helix

* In certain materials there is said to be a "magnetic viscosity," because of which the magnetization continues to vary for an appreciable time after an alteration in field is made and ended. The observations upon this are much confused by eddy-currents, and the question is still under debate.

around the wire from top to bottom, like the frieze of the Vendome Column; the expansion (or contraction) of the material along this line of force requires the wire to twist.

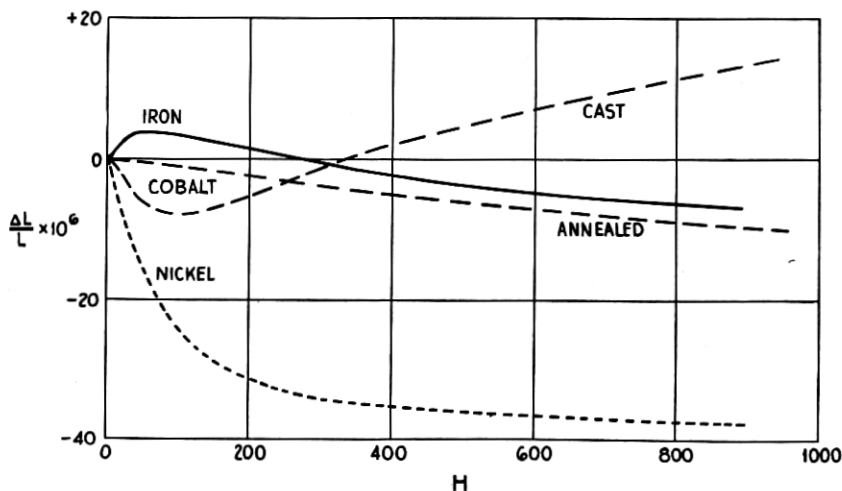


Fig. 4—Magnetostriction (Joule effect) in polycrystalline wires of iron, cobalt ("cast" and "annealed"), and nickel. (After K. Honda and S. Shimizu.)

It is customary to say that, in a gradually-increasing longitudinal magnetic field, nickel contracts continually; cobalt contracts at first, then returns to its original length, then expands; iron first expands, then returns to its original length, then contracts; the Heusler alloys expand continually. Unfortunately, some at least of these statements are valid only for samples which have been and are being treated in particular ways. One finds in the literature, for instance, the information that hard steel and very-well-annealed cobalt behave like nickel, shortening continually as the field is augmented. If the rules which I stated at first are really typical of the respective elements in standard states, then one may lay what emphasis he chooses on the fact that the four consecutive elements which are nickel, cobalt, iron and the manganese which is the essential element of the Heusler alloys are associated each with a different one of the four conceivable permutations of expansion and contraction.

The change in length, whichever its eventual sign, comes to an end when the material is magnetized to saturation. Intensity of magnetization is therefore the natural independent variable on which to consider magnetostriction as depending.

Quite the most exciting of the lately-discovered facts about magnetostriction is disclosed in Figure 4a, which consists of curves representing

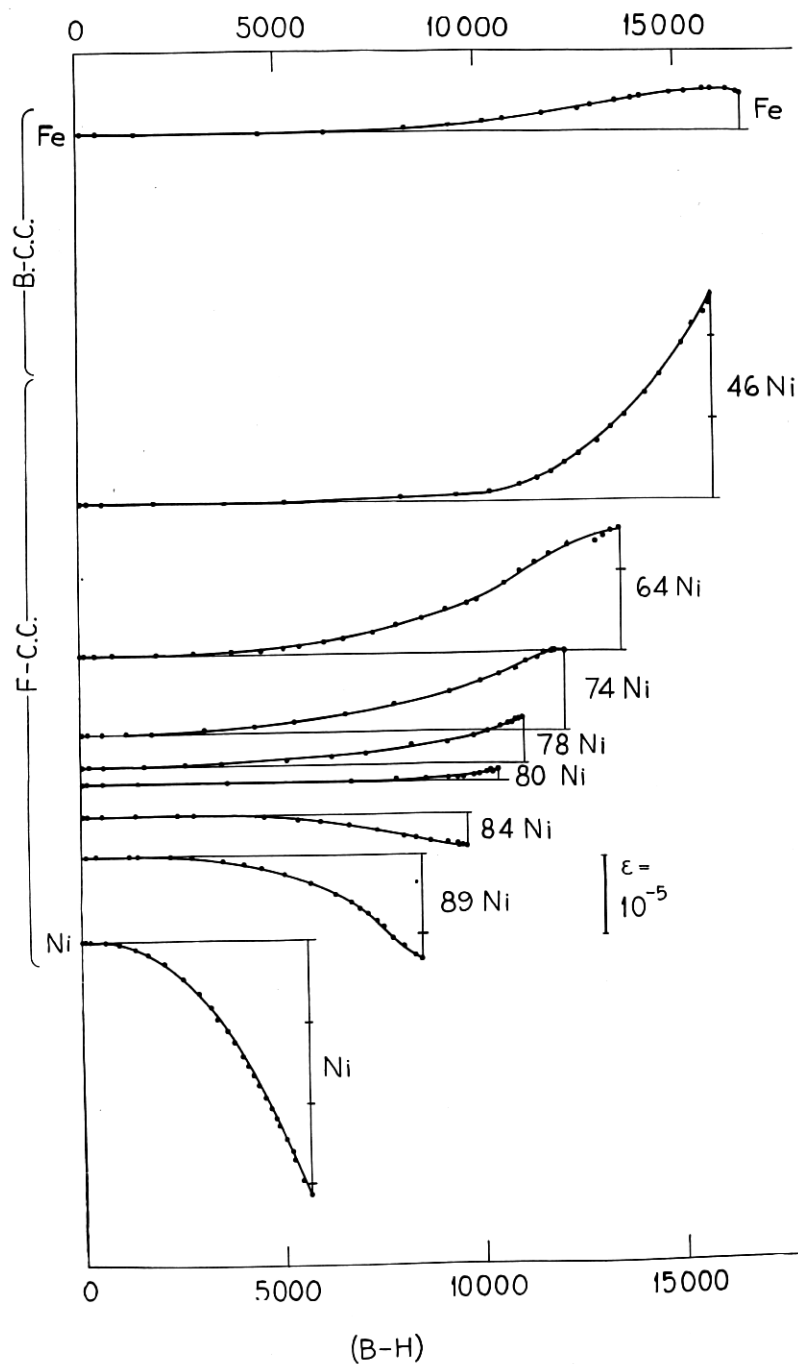


Fig. 4a—Magnetostriction in polycrystalline wires of annealed iron, nickel, and various permalloys. (After L. W. McKeehan and P. P. Cioffi.)

change-in-length for iron, nickel, and six permalloys in which the percentages of nickel are those indicated beside the curves. The term "permalloy," I recall, is applied to iron-nickel alloys containing more than 30 per cent of nickel, of which the initial permeability is remarkably high; the heat-treatments which these alloys had undergone conferred that quality on them, the nickel had been treated in quite and the iron in nearly the same way. The abscissa is intensity of magnetization, for the reason aforesaid; consequently the curves terminate when this reaches its saturation-value (not, however, attained in the experiments on iron and nickel).

These curves display the gradations from a steady lengthening reminding the onlooker of the initial lengthening of iron (not, however, followed by a contraction) to a steady contraction approaching the scale of that which nickel displays. Intermediate there lies an alloy which is influenced very little, indeed up to a high stage of magnetization it suffers no perceptible change at all; and this is precisely the alloy having the greatest permeability and the least hysteresis in the entire series. Upon this correlation McKeehan founded his theory of ferromagnetism.

This series of curves reveals other curious facts; for instance the extreme ineffectiveness of the first stages of magnetization in developing the strain—the 46 per cent-nickel alloy had expanded, by the time it was magnetized one third of the way to saturation, by less than one one-hundredth as much as it was destined to expand in acquiring the remaining two thirds of its final magnetization. This is significant; and more significant yet is the point, that when the magnetic field is applied to one of the permalloys containing less than 80 per cent of nickel and subject to a length-increasing longitudinal tension, the magnetostriction is much reduced—that is to say, the mechanical tension seems to have effected of itself a large part of that task of extension which else it would have been incumbent upon the magnetization to perform. It effects a great deal more, of course; the extension due to even a moderate load is vastly greater than the extension which even the greatest of magnetic fields could by itself ever cause; the point is, that the former extension seems to include the latter. Furthermore, elongation by tension is found to produce just as great and no greater an increase in the electrical resistance of a permalloy wire than the much smaller maximum elongation attending magnetization. Yet tension by itself does not magnetize; hence the change which it produces inside the wire does not entirely overlap the effect of magnetic field. It is also true, as one would expect, that tension acting upon a permalloy containing more

than 80 per cent of nickel so affects it that the magnetostriction is increased—the tension seems as it were to have undone something, which the magnetic field must restore before proceeding to the contraction which it operates upon the unstrained metal.

Effect of Tension upon Magnetization

The effects of strain upon magnetization are very complicated, and one would almost despair of ever being able to interpret them, were there not certain relations between them and the effects known collectively as “magnetostriction”—between, to take the simplest instance, the influence of magnetizing upon length and the influence of lengthening upon magnetization—which indicate that law and order reign even in this seemingly chaotic field.

I mention the simplest instance only. A nickel wire, as we have seen, shortens when magnetized parallel to its length; well, when such a wire is shortened by compression, it becomes more magnetizable, the value of I and the value of I/H produced by a continually-applied field H increase; when it is lengthened by stretching (a much easier, consequently a much oftener performed process!), its susceptibility falls off greatly. An iron wire is lengthened when magnetized a little, shortened when magnetized strongly; when it is lengthened by stretching, the magnetization which a weak field imparts to it is increased, that which a strong field imparts to it is diminished; the magnetization-vs.-field curves for different extensions intersect one another somewhere upon the “second segment.” Again, a cobalt wire, when lengthened by stretching, has a lower susceptibility in weak fields and a higher susceptibility in strong fields than it does when untensed; this corresponds to the rule governing the magnetostriction of cobalt. To the Wiedemann effect there correspond a magnetization which occurs when a wire carrying a current is twisted, and a rush of current which occurs on twisting a wire already magnetized. The signs of these effects, and of various others, vary from one ferromagnetic metal to another, and vary in iron and cobalt when the magnetization is sufficiently varied, in the ways which may be deduced from the corresponding magnetostrictive effects.

The variations in magnetization produced by extension may be very much more striking than the variations in length produced by magnetization. In nickel, for instance, the susceptibility of a wire may be reduced to a tenth of its pristine value by stretching the wire, although the utmost change in length which can be brought about by magnetization is less than one part in ten thousand.

The relations between the influence of magnetization on strain, and the influence of strain on magnetization, have been derived from the laws of thermodynamics. It appears that each of the several effects agrees with the theory insofar as the sign is concerned (for instance, tension applied to a wire which shortens when magnetized should diminish its susceptibility, and does) but not always in magnitude. I have not heard of anyone renouncing the laws of thermodynamics on this account.

Hysteresis plays a great part in the effect of tension on magnetization; if a constant magnetizing field is applied to a wire while the tension is being cyclically varied, the magnetization when plotted as function of extension follows a hysteresis-loop. Also the first application of a load to a wire is likely to make a sudden and violent change in the value of I . Some avoid these troubles, or try to, by shaking the wire continually or by continually applying an alternating magnetic field during the measurements. These introduce further complications. In fact, if all the data that could be assembled concerning the effect of strains upon magnetization were to be sought out, I suspect that "the world itself could not contain the books that should be written."

The Barkhausen Effect

Imagine once more a piece of some ferromagnetic substance, encircled by a magnetizing coil, through which the current is being steadily increased; encircled also by a loop, which is connected to the voltage terminals of an oscillograph, or to some other device which moment by moment records the electromotive force impressed upon the loop by the changing magnetization. This electromotive force, as I have said, is proportional to the rate-of-change dB/dt of the induction, for which the changing of the magnetizing field is responsible. It is a measure of the rate of magnetization of the sample girdled by the loop. The magnetizing field is being increased continuously; were the magnetization also to rise continuously towards its saturation-value, as we should probably expect, the voltage-curve would be smooth. However, when a sensitive oscillograph is used, the curve is a succession of sharp teeth. The magnetization of the sample evidently proceeds by small but sudden jumps. These can be shown—in the most literal sense of the word "to show"—by connecting a telephone-receiver through an audion-amplifier to the loop. Listening at the receiver, one hears a rustling or a crackling sound; it has been compared with rain beating upon a tin roof, also with coal rattling down a chute. Barkhausen discovered the effect in this way.

By increasing the magnetizing field very slowly it is possible to space the peaks in the oscillographic curve, or the clicks in the receiver, so widely that the bigger can be counted. Listening to the separated clicks, van der Pol estimated that the process of magnetizing a cubic centimetre of iron or of an iron-nickel alloy involves several thousand of the jumps. It is also possible to measure the area under each of the larger peaks in a curve obtained with a good oscillograph, and calculate from it the magnetic moment of a magnet, the sudden creation of which within the substance would have resulted in just such a peak. One observed by E. P. T. Tyndall could have come about through the sudden creation of a magnet of moment .0027. The word "creation" must not be taken too literally; it might imply, for instance, that two adjacent magnets were at first pointed contrariwise to one another, and one of them was suddenly wheeled around by the field, so that they ceased to neutralize each other. Data such as that just cited from Tyndall would then indicate the sizes of the magnets preexisting in the substance; data such as those of van der Pol, their number. Both sets of data show that one cannot identify these magnets with individual atoms; they are too large (the moment .0027 is as great as that of a piece of saturated iron 0.12 mm. on a side) and too few. Neither can they be identified with individual crystals; a piece composed of a single crystal makes as much noise in the receiver, while being magnetized, as a fine-grained sample. The data suggest that ferromagnetic metals are built up out of magnetic units larger than atoms and smaller than crystals—a suggestion which to the theorists is often extremely acceptable. It is also a welcome fact, that the peaks and the crackling are associated with the steeply-sloping segments of the magnetization-curve, while the initial and final nearly-horizontal arcs of the curve are smooth and silent.*

Magnetization of Single Crystals

Ferromagnetic crystals large enough to be studied are only just ceasing to be a rarity. Only two sorts occur in Nature: those of magnetite (a modification of one of the oxides of iron, Fe_3O_4) and those of pyrrhotine (a sulphide of iron, Fe_7S_8). To procure single crystals of a metal or an alloy, it used to be necessary to wait on the hazards of the foundry, out of which there might arise at long intervals a single large uniformly-crystallized lump. This condition prevails

* Attention must be drawn to the possibility that the peaks in the curve, or the clicks in the sound, are due to fortuitous coincidences of events individually too insignificant to be perceived. Should this turn out to be the case, the Barkhausen effect would resemble the Schroeteffect of thermionic emission, and the interpretation of the data would be changed.

no longer; there are methods for producing large single crystals of metals at will, whether by direct solidification from the melt or by suitable treatment of the masses of randomly-disposed minute crystals which blocks of metals usually are; and there are methods for determining the orientations of the axes of these crystals by means of X-rays. So lately have these methods been developed (they are outgrowths of researches of the last ten or fifteen years) that the first data concerning the ferromagnetic crystals, except for some relating to magnetite and pyrrhotine and a very few early measurements on iron, are only now appearing. One has at times a feeling that these are the first really significant data, the only suitable foundation for a theory of ferromagnetism; that the properties of a polycrystalline rod or wire or ellipsoid do not form a proper basis for theorizing, not being even a simple average of the properties of single crystals oriented in all directions, but a deformed and distorted average infected by the crowding and the cramping and the squeezing which the little crystals perpetually inflict on one another.

All but two of the well-known ferromagnetic substances crystallize in the cubic system. (The exceptions are pyrrhotine and one modification of cobalt, which conform to the hexagonal system). In cubic crystals, directions parallel to the edges of the cubes, to their diagonals, to the diagonals of their faces, are called the tetragonal, trigonal, digonal axes, or the quaternary, ternary, binary axes respectively; the planes to which these directions are perpendicular are called (100) planes, (111) planes, (110) planes respectively. This is as much of the technical language of crystal analysis as we shall require. Of the three lattices in which atoms may be arranged in a cubic crystal—simple cubic, body centred, face centred—iron adopts the second, nickel and cobalt the third. The iron-nickel alloys containing more than 30 per cent of nickel copy the nickel lattice (the permalloys belong to this class) while those containing less than 30 per cent of nickel imitate iron. Many other metals which are not ferromagnetic have cubic lattices of the second or third type, none at all a lattice of the first; it is therefore futile to look for any correlation between ferromagnetism and the arrangement of the atoms.

When a magnetic field is applied to a crystal, it produces a magnetization which is not parallel to the acting field—to the resultant, I mean to say, of the applied field and that due to the “demagnetizing effect of the poles”—unless this resultant is parallel either to a tetragonal or to a trigonal or to a digonal axis. If we apply a field parallel to the axis of an ellipsoid or a long rod, cut from a single crystal in such a way that this axis is parallel to one of the specified directions,

the I -vs.- H curve mounts much more swiftly than does the normal curve for polycrystalline iron; the first segment is very short, and the second passes into the third while the field is still low. The slope of the first part of the curve, that is to say the initial susceptibility, is greatest when the axis of the rod is a tetragonal axis of the crystal, less if it is a digonal, least if it is a trigonal axis; though the differences (Fig. 5) are not great. This is sometimes expressed by saying that iron is most easily magnetized along the tetragonal axis, less so along the digonal and least along the trigonal. Magnetization curves consisting of three or four straight lines meeting at sharp corners have been observed by two of the recent students of single crystals, but not by two others; I infer they are still debatable. The saturation value of I , whether it be

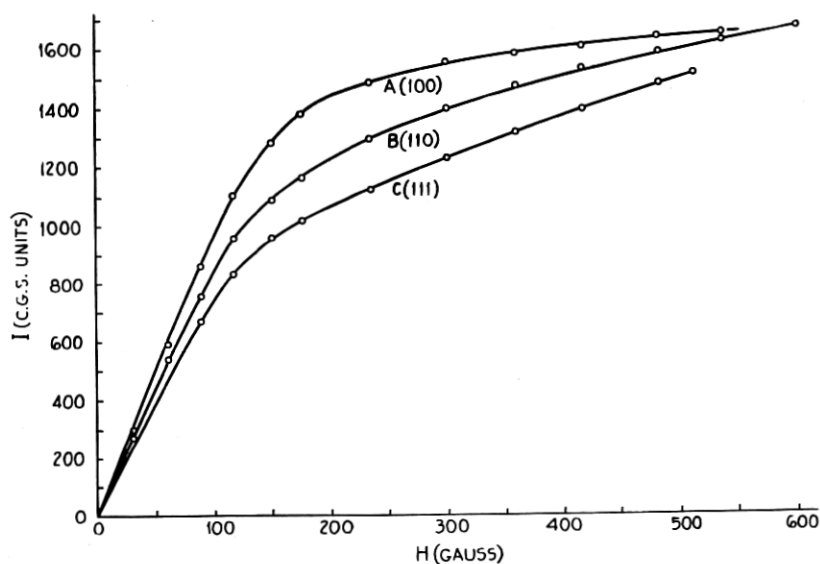


Fig. 5—Initial curves of a single crystal of iron, magnetized parallel to tetragonal (100), digonal (110), or trigonal (111) directions. (After W. L. Webster.)

attained soon or late, seems always to be about the same—another of the reasons for attaching a peculiar importance to it. Honda in fact obtained the value 1707, which he confronts with the 1706 given by Weiss for polycrystalline iron; but this is an agreement which looks too good to be true, or at least to be significant.

The hysteresis-loop for a single crystal is so exceedingly narrow that when it is plotted on any ordinary scale, its sides are too close to be distinguished. Measurements upon rods composed of many crystals, the average size of which varies from rod to rod, show that the area of the hysteresis-loop decreases quite steadily as this average

size of the "grains" is diminished. This is a potent argument against all theories in which hysteresis is attributed to an arrangement of atoms in a uniform space lattice.

When a magnetic field is applied to an iron crystal in any direction not parallel to one of the axes, the magnetization is not quite parallel to the acting field. This manifests itself, for instance, when one cuts a disc out of a crystal and exposes it to a magnetic field in its own

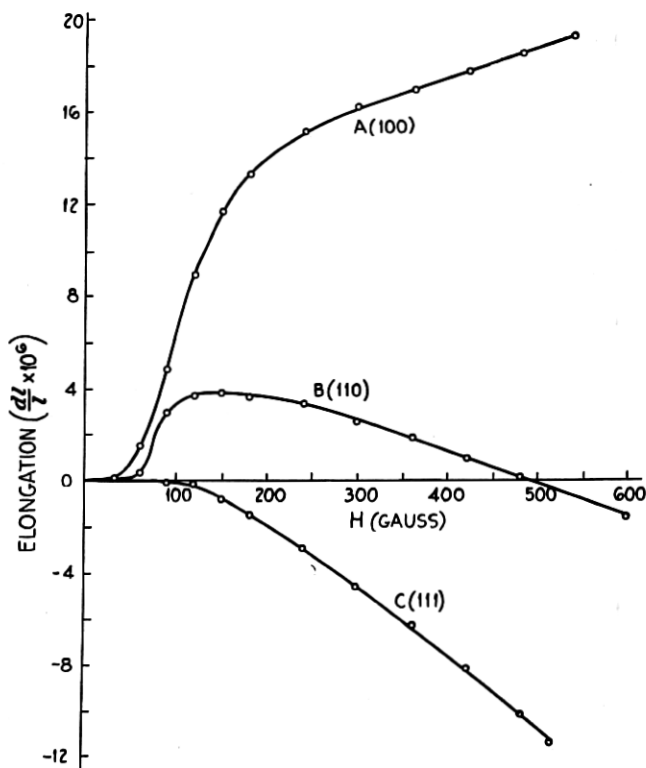


Fig. 6—Magnetostriction of a single crystal of iron, magnetized parallel to tetragonal, digonal, or trigonal axes. (After W. L. Webster.)

plane; it cannot rest in equilibrium until it has so turned itself that one of its three preferred directions lies parallel to the field, for otherwise there is a component of the magnetic moment which suffers a torque from the very field which evoked it. The angle between the vectors \mathbf{I} and \mathbf{H} seldom attains and never exceeds twelve degrees; when the field is kept constant in direction and varied in magnitude, this angle of deviation is less for very weak and less for strong fields than for some intermediate value of fieldstrength. In pyrrhotine,

however, the angle may be enormous—a field inclined at no more than five or ten degrees to the hexagonal axis produces a magnetization which, when investigated by delicate methods, seems to lie exactly in the plane perpendicular to the hexagonal axis, which consequently is known as the “plane of easy magnetization.” A sphere of pyrrhotine to which a bar magnet is brought up from the direction in which its hexagonal axis points does not seem to realize that the magnet is there, but if the approaching magnet is displaced a little

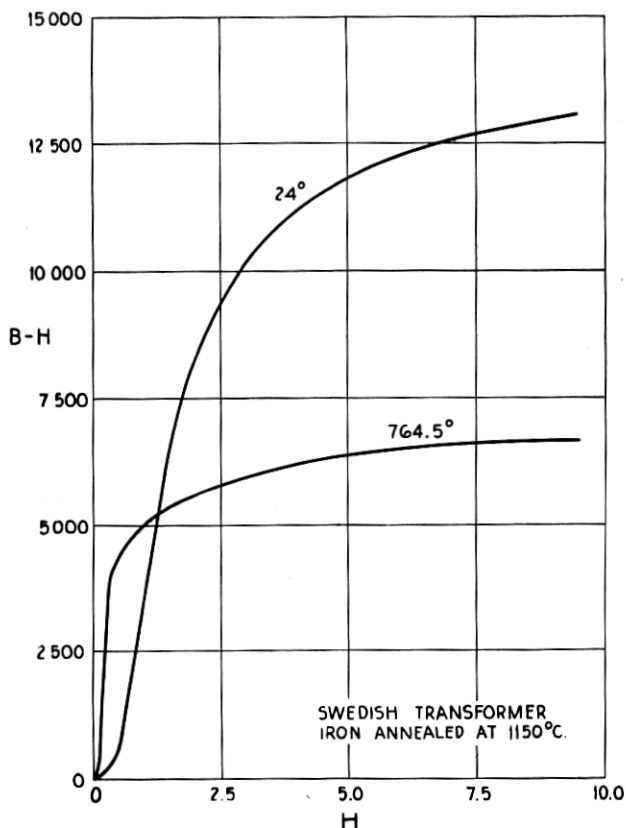


Fig. 7—Initial curves for Swedish transformer iron at two temperatures.
(After D. K. Morris.)

sidewise the ball flies over to its surface at once. It will readily be seen what complications these facts introduce into the mathematics of predicting or describing the magnetization of an arbitrarily-shaped solid body—and in this connection it is well to remember that an ordinary polycrystalline mass of metal partakes as soon as it is strained, by pulling or rolling, of some of the properties of a single crystal.

Magnetostriction in single crystals has some very curious features. A crystal of iron unites in itself all the three modes of magnetostriction which have been supposed typical of iron, nickel and Heusler alloys respectively. A rod having a tetragonal axis along its length expands continually when exposed to a longitudinal magnetic field; a rod cut along the trigonal axis contracts continually; if cut along the digonal axis it first expands, then returns to its original length, finally contracts. The expansion in the first of these cases may attain twenty parts in a million—four or five times as great a value as one ever finds with a polycrystalline sample. This shows how great the extent to which the little crystals in an ordinary block of iron must interfere with one another when the block is magnetized.

Dependence of Magnetization on Temperature

As the temperature of a sample of iron is raised, its normal magnetization-curve varies in a manner suggesting the influence of tension;

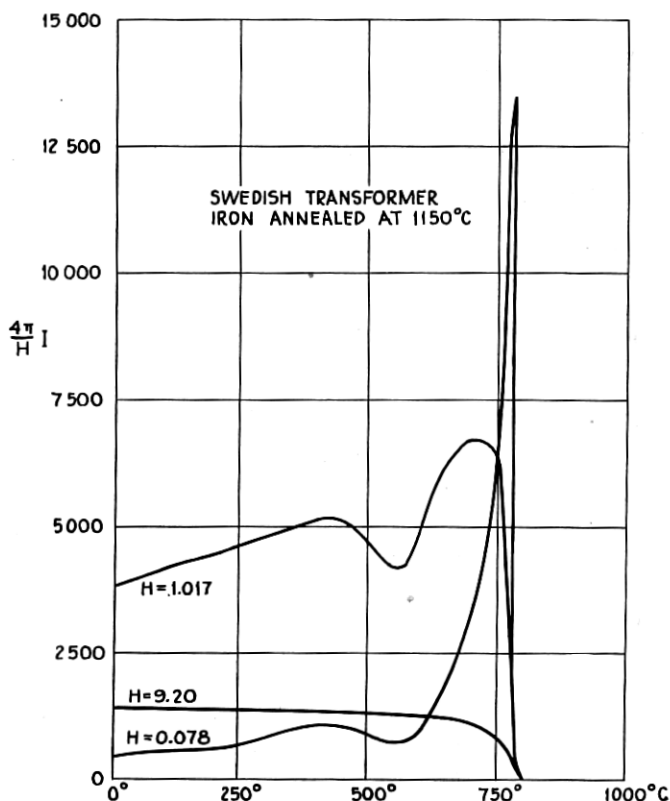


Fig. 8—Magnetization-vs.-temperature curves for Swedish transformer iron at three values of magnetizing field. (After D. K. Morris.)

the earlier part is exalted, the later part is depressed, so that the susceptibility increases in low fields and diminishes in high; curves obtained at different temperatures, not too far apart, intersect one another somewhere upon the "second segment" (Fig. 7). On plotting I or I/H for individual fieldstrengths as functions of temperature, one obtains curves which for very low fieldstrengths, such as 0.3 gauss for instance, are remarkably shaped (Fig. 8). The initial susceptibility rises to an enormous height at a temperature slightly above 700°C. , and then precipitately falls almost to nothing—it does not quite vanish, but instruments of a much higher order of sensi-

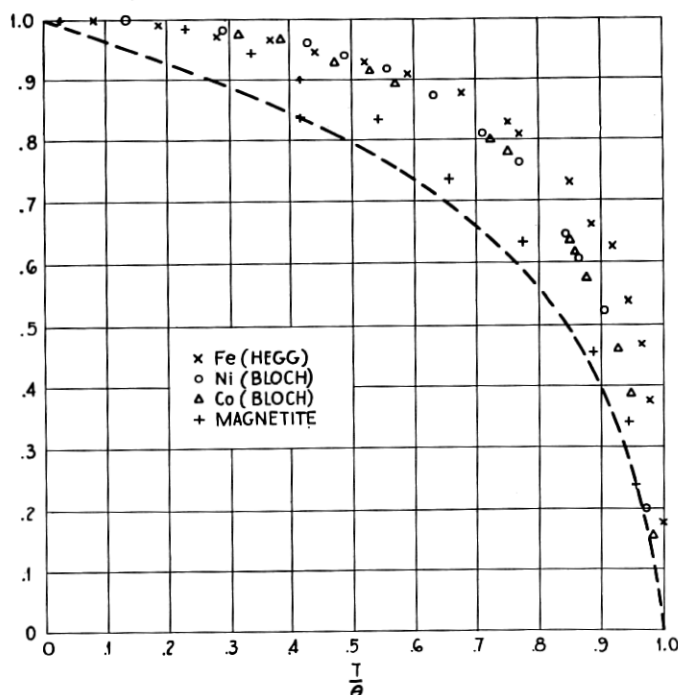


Fig. 9—Saturation-vs.-temperature data for iron, cobalt, nickel, and magnetite below their respective Curie-points, with a theoretical curve. Abscissa is ratio of absolute temperature to Curie-point temperature for each substance individually. (After P. Weiss.)

tiveness are required to detect or measure it beyond say 770° . At a somewhat higher fieldstrength, about 4 gauss, the I -vs.- T curve is nearly horizontal for a long way, and then declines gradually to the axis of H , which it reaches near 770° . At higher fieldstrengths the decline sets in progressively earlier (Fig. 8). At very high fields one obtains what is substantially the curve of $I_{\max.}$ versus T (here the analogy with the effect of tension breaks down) which is shown in Figure 9.

The temperature at which I -vs.- T curves intersect the axis of T , or would intersect that axis were it not that they turn aside shortly before reaching it, is known as the *Curie-point*. For iron, values of Curie-point ranging from 768° to 790° are given; the differences seem to be due partly to uncertainties in deciding just where the I -vs.- T curves "would intersect the T -axis if they continued on downward without turning," partly to the indubitable fact that these intersection-points are not the same for I -vs.- T curves for different values of H , and partly to the use of other definitions of the Curie-point. For nickel, cobalt, and magnetite the Curie-points are in the neighborhood of 360° , 1130° and 550° respectively; and values are recorded for a considerable number of alloys.

The Curie-point is not the sign of what is properly designated as a "change of phase." Iron suffers changes of phase at temperatures near 900° and near 1400° , changes in which the atom-lattice goes over into an entirely different type, and a number of physical properties are sharply altered; but the Curie-point is not one of these, it is the locality of merely a rapid (though not absolutely sudden) change in magnetic properties and an evidently-correlated anomaly in specific heat.* As for the real changes of phase, they normally occur at temperatures so high that they do not influence the magnetization of iron below the Curie-point. Yet it is possible to bring one of the high-temperature modifications suddenly down into the low-temperature range, and then its magnetic properties are quite different from those of "ordinary" iron. In certain alloys this possibility is easy to realize; I will quote only the notorious case of a "nickel-steel" discovered by J. Hopkinson, which at 580° C. is merely one of the many non-ferromagnetic metals, remains so as it is cooled all the way down to zero, then turns suddenly into a modification which is strongly magnetizable and retains this state as it is being heated all the way back to 580° C. But indeed ferromagnetism of alloys is entangled with all the infinite complexities of the behavior and the internal changes of these complicated substances, and varies with all the variations of the more or less durable equilibria between their components.

Definition of Ferromagnetism

Ferromagnetism has sometimes been defined as "the kind of magnetism which iron exhibits"—an easy evasion, to which one is

* Contrary statements about iron are to be found in the early literature; but they are due partly to inaccurate experiments, and partly to the fact that the change-of-phase which in pure iron lies well above the Curie-point descends when carbon is progressively added to the iron, and before long comes into coincidence with the Curie-point; and if still more carbon is added, the "vanishing of ferromagnetism" takes place at the transition temperature.

sorely tempted to have recourse after the first few efforts to devise a better definition. Let us, nevertheless, at least take notice of a few of the alternative proposals.

Materials are classified into diamagnetic and paramagnetic and ferromagnetic. To distinguish those of the first sort is relatively easy, since in any of them the magnetization I called forth by an applied field H is antiparallel to H (in isotropic materials, at least; in crystals the angle between I and H lies between 90° and 180°). In materials of the second or of the third sort, the vectors I and H are parallel and point in the same sense, or at least are inclined to one another at angles smaller than 90° —provided, that is to say, that the material was demagnetized before H was applied. To distinguish between paramagnetic and ferromagnetic bodies, therefore, we must seek some other criterion.

The magnetization of iron, nickel, cobalt, certain of their alloys with one another and with other metals, and the Heusler alloys, may attain values enormously greater than those which can be impressed upon other substances with the highest possible fieldstrengths. One might therefore select some intermediate value for I , and say that all substances for which I may surpass this critical value are ferromagnetic, all others paramagnetic (or diamagnetic). In practice this is usually convenient, because of the great contrast between the substances which I just listed and practically all others. Among the elementary metals apart from the iron-cobalt-nickel triad, one of the most magnetizable is platinum, which shares a column of the periodic table with that triad; yet its susceptibility is only $2 \cdot 10^{-5}$, and an applied fieldstrength of 20000 gauss would impart to it a magnetization of less than one unit, which is utterly negligible compared to those which are easily imprinted even upon the less magnetizable of the substances which I named. The contrast is therefore great enough to be the basis for a useful definition. Yet it must be regarded as accidental, that in practice we are nearly always confronted with extreme cases of one sort or the other. If we travel along the iron-manganese or the nickel-chromium series of alloys (to take but two instances), or if we follow pure iron through a sufficient range of rising temperatures, we find a continuous series of intermediate stages between one extreme and the other; and in principle it is necessary to take account of these.

The magnetizations of iron, cobalt, nickel, certain of their alloys and the Heusler alloys increase, when the applied field is continuously increased, in the curious ways which I described above, attaining maximum limiting-values at fieldstrengths well within the practi-

cable range; while with nearly all other materials I is apparently proportional to H as far as the field can be carried. Here again there is a contrast so great that it can serve as the basis of a useful distinction. Yet all the intermediate stages between the two extremes are exhibited by iron at the various temperatures between 700° and 800° C. Furthermore, there is reason from theory (as we shall see) for supposing that the magnetization of any substance would cease to be proportional to H and would approach a limit, if we could force the field to high enough or the temperatures to low enough values. In fact, there is at least one of the substances conventionally called "paramagnetic" (it is gadolinium sulphate) for which I was found to approach a limit, when the applied fieldstrength was increased while the substance was maintained at the unprecedentedly low temperature of 1.93 K. It is therefore evidently something of an accident that in practice we nearly always meet either with substances for which the ratio I/H is constant within the accuracy of measurement throughout the feasible range of the fieldstrengths, or else with substances for which that ratio varies greatly and unmistakably with the field.

Presence or absence of hysteresis is the third and last of the usual criteria. Iron and cobalt and nickel and some of their alloys and the Heusler alloys exhibit hysteresis-loops, and residual magnetism, and coercive force; and the normal magnetization curve must be distinguished from curves obtained by other procedures for varying the applied field, and one must bother with demagnetizings or else take account of the prior magnetization of whatever sample he is working with. Other substances are free from these complications. Here also it is probable that in iron all the measurable features of hysteresis dwindle off continuously to zero as the metal is heated. On the other hand, it appears that gadolinium sulphate, in spite of acquiring a curvature in its I -vs.- H curve at extremely low temperatures, does not acquire hysteresis and residual magnetism. Perhaps, then, it is better to take the presence of hysteresis rather than the inconstancy of the ratio I/H as the sign of that curious quality, whatever it may be, which makes iron notable among metals.

The general conclusion seems to be the same, as for many other classifications—that is to say: It is possible to draw distinctions between "ferromagnetic" and "paramagnetic" substances, valid for extreme cases of the two types, not sharply marked for intermediate cases; but it happens that for the time being the intermediate cases are in practice not conspicuous; and consequently the distinctions—any one of the three which I mentioned—are useful and worth the making.

C. THEORIES OF FERROMAGNETISM

To devise a theory of ferromagnetism is not necessarily the same task as to make a theory of magnetism. In studying the properties of paramagnetic and those of diamagnetic bodies, one finds many indications that the ultimate atoms of the elements are magnets of definite and seldom-changing moments, or at least may profitably be so regarded. The theory of line-spectra reinforces this opinion, and it is confirmed by the observations of Gerlach and Stern upon the deflections undergone by free-flying streams of atoms traversing a strong magnetic field with a strong field-gradient.* Now, to say that atoms are magnets is scarcely tantamount to giving an explanation of magnetism. On the contrary, the problem is merely pushed a step further away, and must eventually be faced again and either be solved by explaining why atoms are magnets, or else be given up by conceding that magnetism is one of the fundamental properties of matter. Yet it is quite logical and sensible to aspire to construct a theory of *magnetization*—of the gradual magnetizing of a substance by an increasing applied field, of the shapes of the I -vs.- H curves, of hysteresis-loops—out of the assumption that the ultimate atoms are permanent magnets. To explain the gradual rise of an I -vs.- H curve by postulating atoms which are already magnetized to saturation, to explain hysteresis by postulating atoms which individually have no hysteresis—these would be triumphs not open to the objection made against many “explanations,” that they are achieved by ascribing to the atoms the very properties to be explained.

We shall presently make the acquaintance of “elementary magnets”—hypothetical beings, of which each magnetizable substance is supposed to consist. To these we shall assign, for the time at least, definite and unchangeable magnetic moments. A magnetic field applied to an assemblage of such magnets could not change the moment of any. Yet it could change the net magnetic moment of the assemblage, which is the resultant of the moments of all the individuals; for it could, directly or indirectly, cause the elementary magnets to align themselves along its own direction. The assemblage, the substance, would be magnetized *not through magnetization of the individuals but through orientation of the individuals* which make it up.

That idea is an old one; but by itself it is nearly useless. We must think of some agency which could combat the tendency of the elementary magnets to align themselves along the field; for there must be such a one, as otherwise the weakest possible field would magnetize each substance to saturation; which is not the case. The most

* I refer for these to my *Introduction to Contemporary Physics*, pp. 48–50, 383–393.

celebrated theories of magnetization rest upon speculations about the nature of this agency which fights against the field.

On considering the unsurpassably simple system composed of *only two* elementary magnets close together, J. A. Ewing discovered that their interactions are such, that they can prevent each other from aligning themselves immediately along the field; one can almost say that they "interlock," and they interlock in such a way, that the pair of them displays a tripartite I -vs.- H curve, and the quality of hysteresis, though neither separately has any such properties. Systems comprising a dozen, a score, or a multitude of such magnets, arranged in chains or in a cubical array, may be devised to imitate actual initial curves and actual hysteresis loops with stunning accuracy (Fig. 11). Such close agreements need not be overstressed. The astonishing feature of Ewing's discovery is (I think) that although each individual magnet possesses neither the quality of gradual magnetization nor the quality of hysteresis, a pair of them put close together possesses both. So great a result is attained from so simple an apparatus, that it seems very unlikely that any radically different explanation of either quality will ever be put forth. Whatever may be added to Ewing's model, its central idea will probably never be supplanted.

P. Langevin, devising a theory for paramagnetism, supposed that the agency which combats the aligning influence of the field is the thermal agitation of the magnetic atoms. Contrary to one's first impression, this theory is not easily visualized; but it establishes a union between paramagnetism on the one hand, and the great general principles of thermodynamics and equipartition of energy on the other. In the form in which Langevin put it forth, it does not account for hysteresis.

P. Weiss supplemented Langevin's theory by supposing that the actual magnetizing field prevailing inside a magnetizable substance is not that sum of the applied field H_e and the "demagnetizing field" H_i which I defined in Section A, but a combination of this sum with another term depending on the magnetization. As I stressed in Section A, experience teaches us nothing about the value of the true field inside a magnet; Weiss' assumption was therefore a perfectly legitimate choice, to be justified (if at all) by its fruits. One of these is, that it accounts for the presence of hysteresis at low temperatures and its absence at high.

Ewing's Theory

Ewing conceived a piece of iron as an assemblage of tiny bar-magnets, each endowed with a fixed and constant magnetic moment, and wheeling about a pivot under the combined influence of the

impressed magnetic field and the magnetic attractions and repulsions of its neighbors.

Imagine a chain of long slender bar-magnets end to end, the positive pole of each almost touching the negative pole of the next—that is the equilibrium position which they would naturally assume, so long as no external field affects them. By preference, build such a chain out of pivoted magnets; for Ewing's model enjoys the singular merit, that it can be made out of actual magnets and exhibited to the eye. Now there is a remarkable feature of this chain: if a magnetic field is applied to it in some oblique direction, then so long as the fieldstrength is quite small the individual magnets incline themselves toward it slightly, each setting itself at the same angle to the direction of the chain which was originally the common direction of them all; and when the fieldstrength is gradually increased the angle increases gradually, but only up to a certain point—for suddenly, at a critical moment, all the bar-magnets very suddenly capsize, and set themselves in nearly the direction of the field. I use the word *capsize* to invoke the too-familiar analogy of the upsetting boat. As weights are piled upon one side of a boat, it responds at first by tilting gradually sidewise and downward; to each slight increment of the load it accommodates itself by finding an equilibrium-slant a little farther over; but eventually there comes a moment when balance and compromise are no longer possible; the boat cannot find a position of equilibrium except by overturning, and this it does, suddenly and irrevocably. Such is the behavior of a chain of bar magnets; and this is the property which adapts it for representing the general shape of an initial magnetization-curve such as I showed in Fig. 1, with its first slowly rising arc followed by the rapid uprush and the final slow adjustment to saturation.

The overturned boat will not right itself even when the load which upset it is removed; will the chain of bar-magnets be equally unfor-giving? The analogy is not perfect, except in one very particular case: if the angle between the direction of the chain (defined as the direction in which the north poles of all the magnets originally pointed) and the direction of the field is 180° , the capsizing will result in a right-about-face of each magnet and a reversal of the so-defined direction of the chain, and this reversal will persist after the field is annulled.

Suppose however that there is a multitude of chains oriented at random, so that half of them are inclined at less than 90° and half at more than 90° to the direction of any strong field which we choose to apply. The field will cause all the bar-magnets to capsize (except those belonging to the few chains to which it is almost parallel);

and thereupon, those which belong to the chains originally inclined at more than 90° to the field are more than halfway turned around, and when the field is nullified they will realign themselves with their first associates, but every one will be reversed. Originally the net magnetization of the assemblage of chains was nil, for half neutralized the other half; now it is considerable, for half have been inverted. Its ratio to the total magnetization of all the chains when parallel is, in fact, one half. This consequently would be an adequate model for a substance of which the remanence is one half of the saturation-intensity.

Other values than one half for the ratio of remanence to saturation can be derived from Ewing's picture by choosing a suitable arrangement for the elementary magnets. Suppose, for another and a final example, that they are arranged in a cubic lattice, so that each has the choice (as it were) of orienting itself along any one of the directions parallel to the cube-edges. Chains of magnets may then form themselves along any one of six possible directions (counting the two opposite senses of any line parallel to a cube-edge as two distinct directions). In a demagnetized crystal, one may imagine that the elementary magnets in the lattice fall into groups or "complexes," within each of which all the chains are parallel, while from one complex to the next they change over from one to another of the six specified possible directions. In a demagnetized piece of metal composed of many small crystals oriented quite at random, there will be chains of magnets pointing in all directions. To such a piece of metal let a field be applied, increased to so great an amount that it saturates the material, and reduced gradually to zero. Whatever the direction of the field, it will be inclined at 45° or less to one or more of the six possible directions for the magnet-chains in every crystal. As the field is varied in the manner which I have described, the magnets in each crystal will be wheeled into parallelism, and subsequently will relapse into chains pointed in that direction (or those directions) which makes the least angle with the field. The ratio of remanence to saturation for a polycrystalline sample, resulting from this model, should then be 0.893.

By adjusting the disposable constants, Ewing's model may be made to predict not only the general shape of the I -vs.- H curve, but the values of fieldstrength and magnetization at which the first segment of the curve should pass into the second. Apparently no very pleasing agreements between experiment and theory have yet been attained in this way. Nevertheless I will show how the attempt is made; by doing so, I can at least bring out the influence of the various disposable constants upon the result.

The simplest form of Ewing's model* is composed of linear chains of elementary magnets. To analyze this it suffices to consider a system composed of two identical magnets, each so long and slender that it may be visualized as a pair of poles of equal polestrength M separated by the length L of the magnet, and both of them pivoted around their centres at points distant from one another by a spacing S which is only slightly greater than L (Fig. 10). If there is no external field, they come to an equilibrium, in which state both point in the same sense along the line of centres. If there is an applied field oblique to the line of centres, they come to an equilibrium in which both are deflected through equal angles from that line. Denote their angles of deflection by θ , the angle between the field and the line of centres by α , the fieldstrength by H , the distance between the adjacent unlike poles of the magnets by R . The distance R is equal to $(S - L)$ when θ is zero, and in general is given by the equation:

$$R^2 = L^2 + S^2 - 2LS \cos \theta. \quad (1)$$

The adjacent poles attract one another with forces M^2/R^2 directed along R , which result in torques T' upon each magnet:

$$T' = \frac{M^2 L}{R^2} \sin \varphi = \frac{M^2 L S \sin \theta}{2R^3}. \quad (2)$$

The remote poles likewise exert forces upon the adjacent poles and upon one another, and torques upon the magnets; but it will be necessary to reduce these to relative insignificance by supposing the

* The next four pages resulted from an attempt to formulate what I take to be Ewing's objection to his own early model, which he phrases in these words: "Now it is known that in ordinary iron barely one per cent of the whole magnetism of saturation is acquired in the quasi-elastic stage before the effects of hysteresis set in. To conform to this condition the magnets of the model must have only a very narrow range of stable deflexion, and consequently they have to be set very near together with the result that in the old model their mutual control became excessive. A calculation of the force required to break up rows of pivoted magnets, of atomic dimensions, when set near enough together to satisfy the above condition, showed it to be many thousands of times greater than the force which is actually required, in iron to reach the steep part of the curve."

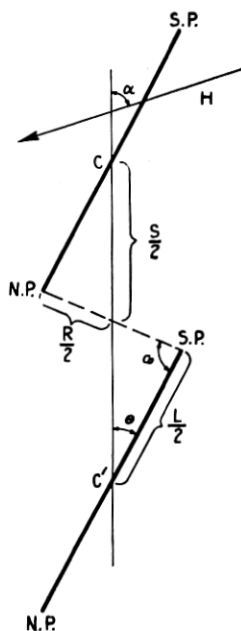


Fig. 10 — Illustrating the elementary magnet-pair of Ewing's theory.

"clearance" ($S - L$) between the adjacent ends of the magnets to be extremely small by comparison with S and L , and by considering only values of θ which are so small that R itself remains small by comparison with L ; otherwise the equations will be hopelessly intricate, and they are more than bad enough even with this restriction. Happily the model possesses some of the required properties even when limited by this restriction.

The torque T exerted by the field H upon either magnet is given by

$$T = MHL \sin (\alpha - \theta). \quad (3)$$

The general condition for equilibrium is

$$T - T' = 0. \quad (4)$$

The special condition for "neutral" or "labile" equilibrium, i.e. for the state of incipient capsizal, is

$$d(T - T')/d\theta = 0. \quad (5)$$

The values of H and θ , obtained by solving (4) and (5) as simultaneous equations, are the fieldstrength just sufficient to produce capsizal and the angle of deflection attained just before the overturn; they are obtained as functions of the variable α , and of the constants M , L , S which are features of the model.

Solving these equations, however, is easier said than done; they prove surprisingly intractable. Only in one particular case is the solution easy: we must choose values of α so near to 90° , and suppose the clearance and consequently the deflections so small, that the cosine of $(\alpha - \theta)$ may be set equal to zero. In this case equation (5) is reduced to the form

$$dT'/d\theta = \text{const. } d(\sin \theta/R^3)/d\theta = 0, \quad (6)$$

which, if we write a for S/L , is found equivalent to

$$(a^2 + 1 - 2a \cos \theta) \cos \theta = 3a \sin^2 \theta. \quad (7)$$

Putting $a = 1 + \epsilon$ —so that ϵ stands for the quantity $(S - L)/L$, which by hypothesis is small—and neglecting powers of ϵ higher than the second, we arrive at the equations:

$$\cos \theta_c = 1 - \frac{1}{4}\epsilon^2; \quad \sin \theta_c = \epsilon/\sqrt{2} = (S - L)/L\sqrt{2}, \quad (8)$$

for the value θ_c of the deflection just at the verge of capsizal; and putting this into equation (4), we get

$$H_c = \frac{M}{3\sqrt{3}(S - L)^2}. \quad (9)$$

Equation (9) gives the fieldstrength H_c which effects capsizal of a pair of magnets initially transverse to the field, and having a clearance $(S - L)$ extremely small compared with their lengths L . For a chain of magnets the value given for H_c would need only to be doubled; for any number of chains lying in the plane normal to the field, that double value of H_c would remain valid. It is, we see, proportional directly to M and inversely to the square of the clearance.

Multiplying the expression given in (8) for $\sin \theta_c$ by ML , we get the component along the field-direction of the moment of any magnet belonging to such a pair or to such a chain. If there were N magnets grouped in pairs or chains in the plane normal to the applied field, the magnetization I of the entire assemblage, parallel to the field-direction, would be $NML \sin \theta$. The magnetization I_c at the verge of capsizal

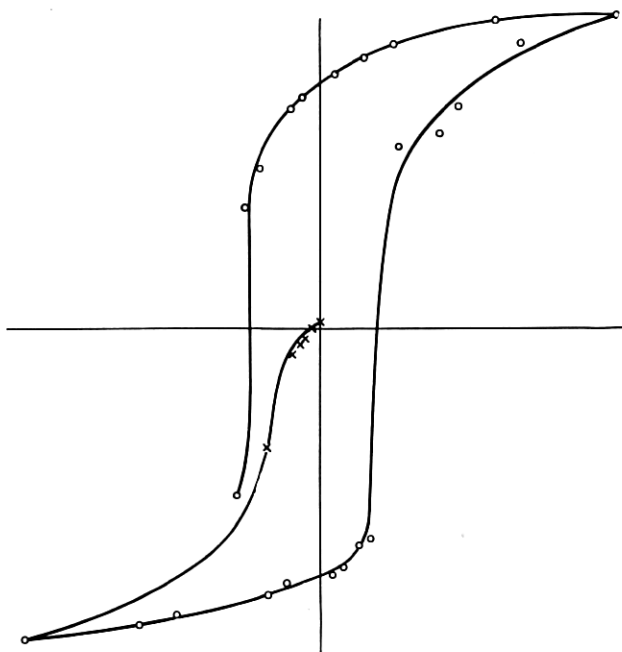


Fig. 11—Initial curve and hysteresis loop of an Ewing model composed of 24 pivoted magnets. (After F. A. Ewing.)

would be $NML \sin \theta_c$. The saturation-value of magnetization, I_{\max} , would be NML . Thus we arrive at the equation for I_c :

$$I_c = I_{\max} \frac{S - L}{L\sqrt{2}}, \quad (10)$$

beside which we may place equation (9), reshaped and with allowance for the doubling:

$$H_c = \frac{2I_{\max}}{3\sqrt{3}NL(S-L)^2} \quad (11)$$

It is now obvious that, in the case of a material for which I_{\max} is known, we have apparently three disposable constants N , L , and $(S-L)$. However, the ratio of $(S-L)$ to L must be a very small fraction; otherwise the assumptions from which the equations were deduced would not be valid. This ratio is determined by the ratio I_c/I_{\max} . If we take for I_c the value of magnetization somewhere near the division between the first and second segments of the initial curve, we find $I_c/I_{\max} = .01$ for soft iron (I quote Ewing) or about .05 for the permalloy of which the curve is exhibited in Fig. 1. Now we have the ratio of $(S-L)$ to L fixed, and ostensibly two disposable constants left. However, if we assume that each elementary magnet is an atom and each atom an elementary magnet, both of these are determined by the crystal lattice of the metal. Nothing remains adjustable; a definite value is imposed by the theory upon H_c . This value is enormously too great.

It is clear that the situation could be saved by dropping the assumption that every atom is a magnet, so that the constants N and L might again become freely disposable. Ewing proposed another way of escape—a modification of the model involving the introduction of a fourth constant. He invented a system composed of three magnets with their centres in a line, the two outer ones fixed and pointing in opposite senses along the line of centres, the middle one free to revolve. The polestrengths of the outer magnets, M' and M'' , are supposed to differ slightly; then, when no outside force is acting, the middle one comes to an equilibrium in which it points in the same sense as the stronger of its neighbors. When a field is applied in a direction inclined at α to the line of centres and steadily increased, capsizal occurs at a certain value of fieldstrength H and the corresponding value of deflection θ . When the clearances are small and α is very nearly 90° , the equation for θ is equation (8) with an unimportant change in numerical factor; while the equation for H is changed, in that $(M' - M'')$ now stands in the place of M . This is the new constant introduced into the model.

Ewing supposed that the pivoted magnet of his model might be the analogue of an internal electron-orbit of the iron atom, while the fixed neighbors might correspond to external distributions of whirling electrons, in the periphery of the same atom or in neighboring

ones. This notion is endangered by the discovery that a single crystal of iron displays only a slight degree of hysteresis, much less than a polycrystalline mass—a discovery which likewise weakens the force of calculations of remanence based upon the assumption of a cubic lattice, such as I gave earlier. In fact, it seems quite probable that in the course of assimilating the newly-acquired data concerning single crystals, all of the numerical agreements hitherto derived from Ewing's and other theories of ferromagnetism may be swept away.

Nevertheless the basis of Ewing's theory is likely to persist; for it has two great advantages which are nearly independent of numerical agreements. Hysteresis is derived from an atom-model in which nothing of the nature of hysteresis is introduced by postulate; and the general effect of mechanical and electrical jerkings, bumpings and joggings is explained in a way which seems most natural and plausible to our mechanical instincts. As for the first point: to explain hysteresis by the mutual interactions of magnets which in themselves are constant and do not possess it is so eminently satisfactory a solution that any theory or model in which hysteresis is introduced *ab initio* or derived from some extra assumption will start under a great handicap. As for the second: to take one illustration, it is well known that a demagnetized piece of iron exposed to a weak field, and endowed thereby with the moderate magnetization corresponding to some point or other on the first segment of the initial curve, becomes enormously more intensely magnetized when it is jolted or jerked. Visualized by Ewing's model, this seems the most natural thing in the world: the elementary magnets which were on or close to the verge of capsizing are pushed over that verge by the mechanical shock. Equally natural seem the annulment of the residual magnetism of a piece of iron, by mechanical shocks and jerks; the like effect of rapidly-alternating magnetic fields; the tendency of a current along an iron wire to favor magnetization of the wire; and the fact to which I alluded earlier, that in a very strong rotating magnetic field a piece of iron does not grow hot, and consequently there can be no hysteresis-loss. This last-named feature may be visualized by supposing that the chains, having been once completely broken up, do not form again as the magnets are whirled round and round. It seems natural also to expect that as the temperature is raised, the chains will be broken up by thermal agitation, and the reversible first segment of the initial curve will mount more sharply and continue longer. In trying to deal with the effect of temperature, however, we soon reach the limits to which Ewing's theory can be forced; and another method of attacking the problem of ferromagnetism recommends itself.

Weiss' Theory

There is another theory of magnetization, built upon an entirely different basis from Ewing's—a basis involving the notion and in fact the definition of temperature. To import temperature into theories of magnetism is clearly most desirable, considering how great is the influence of that variable upon the I -vs.- H curves; an influence so great, indeed, that when a sample of any ferromagnetic substance is made sufficiently hot, all the distinctive features of ferromagnetism depart from it. In developing Ewing's model, it is easy to say that as the temperature is raised the little magnets are more vigorously agitated, the bonds which are responsible for remanence and coercivity are more frequently ruptured; but such statements, though plausible, lack precision and hold out no promise of numerical agreements between theory and experience. That being the case, it seems unreasonable to expect numerical agreements from a theory offering a much less definite and specific picture of the interior of a ferromagnetic body than even Ewing's. Such agreements, nevertheless, emerge from the theory of Langevin and Weiss.

Langevin took as his point of departure the theory of temperature developed by the great savants Maxwell and Boltzmann (the same from which, by the way, the quantum-theory arose through the modifications made by Planck). To introduce as much, or as little, of this theory of temperature as is required for our present purpose, we envisage a sample of oxygen gas, N molecules per unit volume, in thermal equilibrium at absolute temperature T . Let each molecule be visualized as a rigid body of mass m , having three principal axes of rotation and corresponding moments of inertia I_1, I_2, I_3 . The molecules are darting to and fro, with translatory velocities which may be specified by giving the three components u, v, w of each in some coordinate-frame. They are likewise revolving, with angular velocities which may be specified by giving the three components r, s, t of each along the principal axes of the molecule in question. The kinetic energy of the molecule is given by

$$\begin{aligned} K &= \frac{1}{2}mu^2 + \frac{1}{2}mv^2 + \frac{1}{2}mw^2 + \frac{1}{2}I_1r^2 + \frac{1}{2}I_2s^2 + \frac{1}{2}I_3t^2 \\ &= K_u + K_v + K_w + K_r + K_s + K_t, \end{aligned} \quad (1)$$

each of which six terms may be regarded as the kinetic energy associated with the variable which its subscript denotes. We will further suppose that each molecule is a magnet of moment M . When the gas is pervaded by a magnetic field H , each molecular magnet has a potential energy V_θ given in terms of the variable θ , the angle which its axis makes with the field, by the equation

$$V_\theta = -MH \cos \theta. \quad (2)$$

I propose now to show that Langevin's theory of magnetization is obtained by applying to the potential-energy term V_θ the same mode of reasoning as is customarily and familiarly applied to the kinetic-energy terms $K_u \cdots K_t$.

It is well known that the average kinetic energy of translation, the average of the sum of the terms K_u and K_v and K_w , taken over all the molecules of a gas of absolute temperature T , is proportional to T ; it is, in fact, given by the equation

$$\overline{K_u + K_v + K_w} = \frac{3}{2}kT, \quad (3)$$

in which k stands for the ratio of the gas-constant R to the Loschmidt number N_0 (number of molecules per gramme-molecule).^{*} The average of each of these three terms separately is equal to $\frac{1}{2}kT$; and this result was generalized by Maxwell and by Boltzmann to the three rotational terms in the expression for K , so that

$$\bar{K}_u = \bar{K}_v = \bar{K}_w = \bar{K}_r = \bar{K}_s = \bar{K}_t = \frac{1}{2}kT. \quad (4)$$

We go one step further in the analysis of the motion of the molecules. Consider the distribution-function for any one of these six variables, u for instance; it is given by Maxwell's formula:

$$dN = NC_u \exp(-\frac{1}{2}mu^2/kT)du, \quad (5)$$

in which dN stands for the number of molecules (among the N molecules occupying unit volume) for which the velocity-components along the x -axis lie between the values u and $u + du$. The constant C_u is so adjusted that the integral of dN over the entire range of values of u shall be equal to N ; on being computed it turns out to be $\sqrt{m/2\pi kT}$. The quantity $\frac{1}{2}mu^2$ is the one hitherto designated as K_u . For the distribution-function with respect to u , which is the coefficient of du in (5), and may be denoted by $F(u)$, we therefore have:

$$F(u) = N \cdot \sqrt{m/2\pi kT} \cdot \exp.(-K_u/kT) \quad (6)$$

^{*}The primitive way of deriving (3), reproduced in all elementary texts, is as follows: Imagine a cubical vessel one cm. along each edge containing N molecules; suppose that $N/3$ molecules are moving in lines parallel to each edge, with uniform speed v ; each face is then struck with $Nv/6$ impacts per second, and in each impact an amount of momentum $2mv$ is communicated to the face, so that the average pressure upon the surface is $p = Nm v^2/3$. According to the well-known gas-law, $p = \rho RT/M$ (ρ standing for the density, M for the molecular weight of the gas); hence $Nmv^2/3 = \rho RT/M$, and recalling that $\rho = Nm$ and that $M/m = N_0$ and that $\frac{1}{2}mv^2$ is the kinetic energy K of a molecule, we have $K = 3RT/2N_0 = 3kT/2$. The same result is reached by more sophisticated methods of averaging.

and the distribution-functions with respect to v , w , r , s , and t differ only by the substitution of the appropriate kinetic-energy term for K_u , and (if necessary) of I_1 or I_2 or I_3 for m .

For the distribution-function with respect to θ , we shall write an equation copied after (5), as follows:

$$\begin{aligned} dN &= NC_\theta \exp(-V_\theta/kT) \sin \theta d\theta \\ &= NC_\theta \exp(MH \cos \theta/kT) \sin \theta d\theta. \end{aligned} \quad (7)$$

The constant C_θ is to be so adjusted that the integral of dN over the entire range of values of θ (which extends from 0 to π) shall be equal to N . It turns out that

$$C_\theta = a/(e^a - e^{-a}) = a/2 \sinh a \quad (8)$$

in terms of the parameter

$$a = MH/kT, \quad (9)$$

which we shall use often enough to justify the special symbol for it. The factor $\sin \theta$ in equation (7) requires comment. Imagine all the molecular magnets brought together at a point P , and their axes prolonged until these intersect a sphere of unit radius traced around P as center. The locus, upon this sphere, of the points of intersection of lines associated with magnets inclined at angles between θ and $\theta + d\theta$ to the field is a belt or collar of area $2\pi \sin \theta d\theta$. There are dN of these points, and they are distributed over this belt with surface-density $dN/2\pi \sin \theta d\theta$. By making dN proportional to the product of $\sin \theta$ into an exponential function, we make that surface-density, which is the density-in-solid-angle of the directions of the magnetic axes, proportional to the exponential function itself; and this is what is done.

We proceed to calculate the net magnetic moment of the assemblage of N molecular magnets. Resolving the moment of each, we find $M \cos \theta$ for its component parallel to the field-direction (with the perpendicular component we are not concerned, since the average of its values for all the molecules is obviously zero). Summing the values of these parallel components for all the molecules, we have:

$$I = \int_0^\pi M \cos \theta dN, \quad (10)$$

the symbol I being used for the sum of the parallel components, since this sum is precisely the intensity of magnetization per unit volume defined near the beginning of this article. Remembering (7) and (8), and performing the integration, we arrive at

$$\frac{I}{I_{\max.}} = \coth a - \frac{1}{a} = L(a), \quad (11)$$

the symbol $I_{\max.}$ being used for NM , the total magnetic moment which the assemblage of N molecules would have if they were all directed perfectly parallel to the field.

This function $L(a)$ is represented by the curve of Fig. 12, which departs from the origin with slope $a/3 = MH/3kT$, and bends over

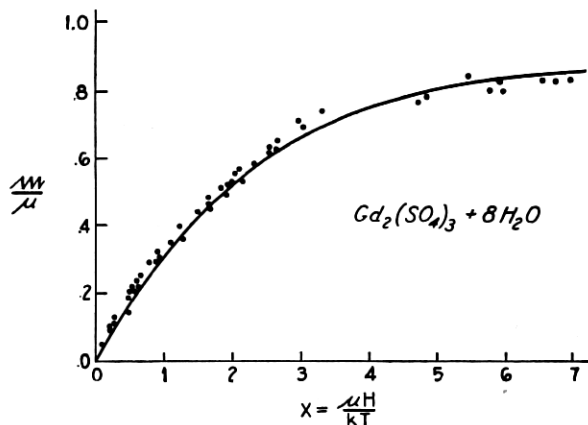


Fig. 12—The I -vs.- a curve of Langevin's theory of paramagnetism, and the data for gadolinium sulphate. (After P. Debye.)

toward its asymptote $L = 1$ without passing through any point of inflection. It has thus a resemblance to the initial curve; but one must not be misled by this, for Langevin's theory is not a theory of ferromagnetism. It is based on assumptions appropriate to a gas, and gases are not ferromagnetic; it gives no account of remanence, and remanence is an essential feature of ferromagnetic bodies. With a gas of which the molecules are permanent magnets, we should expect I to vary with H in the manner indicated by the curve.

Now as a matter of fact, in oxygen and other paramagnetic gases I is apparently proportional to H , up to the greatest fieldstrengths which can be applied:

$$I = \sigma_0 H. \quad (12)$$

This however does not necessarily mean that equation (11) is not valid; it may mean simply that the greatest available fields (some tens of thousands of gauss) are not great enough to pass beyond the sensibly-straight initial portion of the curve. If so, then

$$\sigma_0 = I_{\max.} M / 3kT = NM^2 / 3kT \quad (13)$$

and σ_0 , the susceptibility of the material, should vary inversely as the absolute temperature. This, as Curie found, is true for the paramagnetic gases. It is true also for a number of salts in dilute solutions, and even for a certain number of solid substances, although for these the underlying assumptions would scarcely be expected to remain valid; one has the feeling that the data are left floating in the air by the withdrawal of the logical basis for the theory with which they agree.

Suppose nevertheless that the theory remains valid; then, for any substance of which the susceptibility σ_0 varies inversely as T , one can calculate the moment M of its molecular magnets from (13); for k is a known constant, and N is knowable at least when one is dealing with a gas of known density or a solution of known concentration (with solids there may be doubt as to the number of atoms grouped together to form an "elementary magnet"). Multitudes of such values have been computed; their orders-of-magnitude are 10^{-18} to 10^{-20} . Commonly they are expressed as multiples of a certain unit, the "Weiss magneton," which is equal to $1126/N_0$ or about $1.858 \cdot 10^{-21}$. Many of them are nearly integer multiples of this unit.*

On taking any observed value of M , and multiplying it by the corresponding value of N to obtain the "theoretical" value of I_{\max} . for the substance in question, we find that as a rule this last is so much larger than the highest value of I attained with practicable fields that there is no contradiction between the theory and the fact that I is sensibly proportional to H all through the feasible range of fieldstrengths. There is only one substance (gadolinium sulphate) for which I_{\max} . can be approached and this only at extremely low temperatures, below 5° absolute; in Fig. 12 the data are displayed; it is evident that the Langevin curve, drawn with the initial slope best suiting the points near the origin, fits fairly well to all the other points.

I pass now to the assumption whereby Weiss so extended Langevin's theory that it became competent to describe not only these simplest cases of paramagnetism in which $1/\sigma$ is proportional to T , but also the much more numerous cases of paramagnetic substances conforming to a more general law, and certain aspects of ferromagnetism also.

Formally the extension amounts to this, that in the expression for the parameter a which figures in equation (9), the fieldstrength H is replaced by a linear function of H and I :

* To enter into the long and fiercely debated questions about the meaning and even the reality of the Weiss magneton would lead me too far afield; but it is so frequently used as a unit in stating data of experiment that one must know at least its value.

$$a = M(H + nI)/kT, \quad (14)$$

which is transported bodily into the function $L(a)$ of equation (11). This is a very abstract way of putting the fact; but the more concrete ways have not been satisfying. One may say that the true field acting within the material is not H , but $(H + nI)$ —that the actual though unverifiable field acting at any point in the inaccessible interior of the magnet is the sum of the field H_e due to objects in the external world, and the field H_i due to the “demagnetizing effect of the poles,” and an additional term proportional to the intensity of magnetization at the point in question. The suggestion of Weiss, then, is tantamount to making a new assumption concerning this tantalizing internal field. The natural next step is, to visualize or explain the agent of the extra force, the “molecular field” as Weiss calls it; that is the step which no one has yet succeeded in making, not at least with general assent.

Making the expression in (14) the argument of $L(a)$, we see that the fundamental equation (11) now has the variable I in both its members, and must be solved for I . The resulting function is one of the infinitely many which have neither names nor well-known features, and most of those who write on this subject recommend the high-school expedient of plotting the curves representing the two functions

$$I = (kT/nM)a - H/n, \quad (15a)$$

$$I = I_{\max} L(a) = NML(a), \quad (15b)$$

in a coordinate-plane with I as ordinate and a as abscissa, and looking for the point or points of intersections between the two curves. These, which I shall designate for a few paragraphs as “the line” and “the

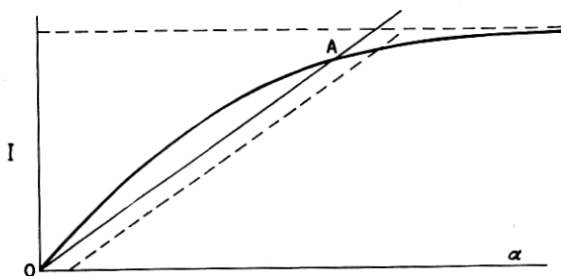


Fig. 13—The “curve” and the “line” of the Langevin-Weiss theory of ferromagnetism

curve,” are shown in Fig. 13. It is easy to see that, when T is held constant and H increases, the line slides from left to right and the intersection-point mounts along the curve; when H is held constant

and T increases, the line wheels counterclockwise around the point where it cuts the axis of abscissæ, and its intersection with the curve descends along the latter.

There is a valuable approximation, which is more nearly valid, the higher the temperature and the lower the field. At the origin, the tangent to the curve ascends with slope $NM/3$ (as I have said) and so long as a is not greater than unity, the ordinate of the curve agrees within six per cent with the ordinate of the tangent. If H is so small and T so great that the crossing of the line and the curve occurs within this range, the problem of locating it may be translated for all practical purposes into the algebraic problem of solving the simultaneous equations

$$I = (kT/nM)a - H/n, \quad I = NMa/3, \quad (16)$$

achieving which, one obtains

$$I/H = \sigma = \frac{C}{T - \Theta}, \quad \begin{cases} C = NM^2/3k \\ \Theta = nC. \end{cases} \quad (17)$$

The susceptibility of an assemblage of elementary magnets, in thermal equilibrium under the influence of an applied field on which there is superposed an extra field proportional to the magnetization of the assemblage, should then depend on temperature approximately

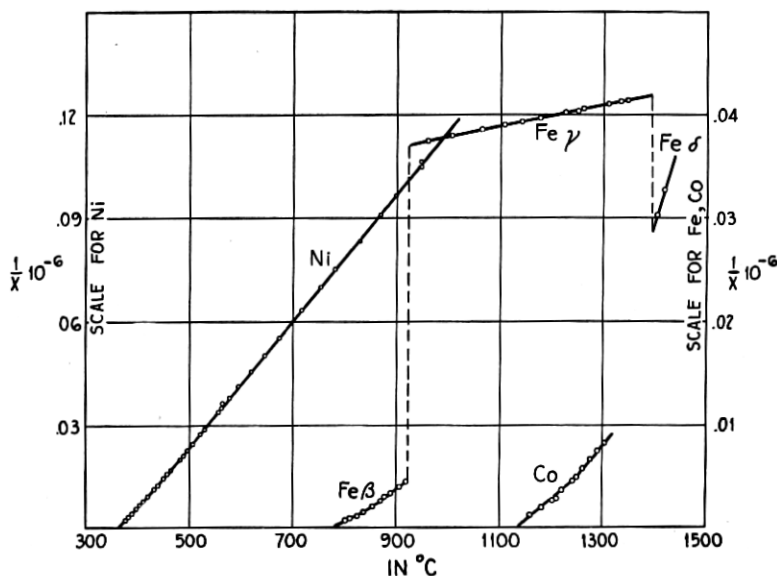


Fig. 14—Susceptibility-vs.-temperature curves for iron, cobalt, and nickel above their respective Curie-points. (After P. Debye.)

according to (17); the approximation being closer, the higher the temperature and the lower the field.*

Now there is a very large class of paramagnetic substances of which the susceptibilities at low fieldstrengths conform, over wide ranges of temperature, to equations like (17); and what renders the theory important for our present purposes is, that the ferromagnetic metals at high temperatures enter into that class. To make the test for any substance it is best to plot $1/\sigma$ as a function of absolute (or Centigrade) temperature. On doing this for nickel beyond the Curie-point (near 360°C.) one finds a curve which at first is somewhat bent, but beyond 410° passes into a beautiful straight line which continues undeflected to 900° . This line is shown in Fig. 14, together with data for iron beyond its Curie-point at 775° ; among these, the points for temperatures between 920° and 1395° lie along a straight line which is sharply broken off at each end of that interval, being followed beyond 1395° by what seems to be the beginnings of an entirely different line, and preceded before 920° by a series of points which are well fitted by a pair of straight lines connected with each other at 828° . The data for cobalt beyond its Curie-point at 1130° likewise conform to a pair of connecting straight lines.

For each of these straight lines one may compute the values of the constants called C and Θ ; and from these, if one accepts the theory, the values of the moment M of the elementary magnets and the coefficient n of the postulated extra force. In calculating M it is necessary to make an assumption about the number of elementary magnets per unit volume of the metal; assuming that there are as many such as there are atoms, and expressing M in Weiss magnetons, Weiss obtained the values 20.9, 17.4, 28.2 and 7.05 for the four straight lines of iron (in order of increasing temperature); 15.9 and 14.55 for those of cobalt; 8 for the solitary straight line of nickel. All these are of the orders of magnitude customarily found in dealing with paramagnetic gases and salts and solutions. The corresponding values of Θ are 1047, 1063, -1340, 1543; 1404, 1422; and 645. The corresponding values of n (which is the quotient of Θ by C) are of the order of several thousands. The postulated extra field must therefore be supposed enormously greater than the field H , and even the induction is quite insignificant by comparison with it. In one of these cases (and in many others among the paramagnetic salts,

* I should state that formulae of the same type as (17) may be derived without assuming that there is a molecular field, provided that we suppose that the distribution-in-energy of the atoms in thermal equilibrium is governed not by the equipartition-law, but by a quantum-law involving a zero point energy.

and in that of liquid oxygen) it must even be supposed *antiparallel* to the field H ; for Θ is negative, and consequently so is n . Necessities such as these make it hard to accommodate the "molecular field" to what is known or conjectured about the interior of solid bodies.

Since it is necessary to assign several distinct values to the coefficient M in order to explain the behavior of iron over various ranges of temperature, one cannot maintain that the iron atom possesses a constant and characteristic magnetic moment which is the source of ferromagnetism. Any such notion, of course, would have been destroyed by facts already mentioned; but it is useful to know these in addition. Changes in M sometimes coincide with great and striking changes in the condition of the metal; at 920° iron exchanges its body-centered lattice (spacing 2.88A) for a face-centered lattice (spacing 3.60A) which it retains as the temperature rises until 1395° is attained, whereupon it returns to the body-centered-cubic arrangement. These alterations in atom-lattice are attended by changes in the physical properties of the metal, so great that three separate "modifications" of iron were distinguished and named before ever the atom-lattices were known or suspected: β -iron normally existing from the Curie-point to 920° , γ -iron from 920° to 1395° , δ -iron from 1395° upward. By certain processes these modifications may be enabled to survive in temperature-ranges not appropriate to them, but that is too long a story for these pages. Changes in M sometimes occur quite unaccompanied, so far as can be made out, by changes in atom-lattice or other physical features. The variation occurring at 828° in iron is of this type, and so is a mysterious change in nickel which in occasional samples brings about values of M near 9, instead of the usual 8 Weiss magnetons.

We turn to residual magnetism, on its explanation of which every theory of ferromagnetism must stand or fall. It is the supreme merit of the theory of Weiss that residual magnetism figures as a property which substances paramagnetic at high temperatures naturally and gradually acquire, when they are cooled below a certain critical point. We shall see this best by returning to Fig. 13. Begin by imagining the line corresponding to a particular pair of values of H and T ; leave T constant, reduce H steadily to zero; the intersection of curve and line slides down the curve, *reaching the origin if the slope of the line is greater, stopping short of the origin if the slope of the line is less, than the slope of the tangent to the curve at the origin.*

The slope of the line is kT/nM ; the slope of the tangent is $NM/3$; the critical condition is, that these be equal, and this occurs when

$$T = nNM^2/3k = \Theta,$$

i.e., when the temperature assumes the value of that constant Θ which previously entered into our equations. If T is greater than Θ , there should be no residual magnetism. If T is adjusted to be equal to Θ and then reduced gradually to zero absolute, the residual magnetization given from the theory—the ordinate of the point where the curve is intersected by the line of slope kT/nM passing through the origin—increases continuously from zero to its limiting value NM , following the curve traced in Fig. 9. That is the central idea of Weiss' theory of ferromagnetism.

The first of the predictions from the theory which can be put to test is the equality between the temperature at which residual magnetism disappears—the Curie-point—and the constant Θ in the equation (17) for the paramagnetism of the substance beyond the Curie-point. For nickel, the agreement is good: 633° against 645° absolute. For cobalt and for iron, the first short straight line out of the sets of two and four respectively, which are given for these metals in Fig. 14, is so adjusted that Θ agrees perfectly with the Curie-point; its aptness to the plotted data supports the theory.

The next question to be asked is whether the curve of Fig. 9 corresponds to experience. In analyzing this question, one makes the discomfiting discovery that the quantity which was defined as residual magnetization in the theory cannot be identified with the quantity defined as remanence in describing the experimental hysteresis-loops. This results from an imperfection, or at least an incompleteness, in the theory. There is nothing in it to account for the initial curve; there is nothing to account for the gradual increase in I produced by applying a gradually-increasing field to an initially-demagnetized piece of iron, and in fact there is nothing to account for the existence of demagnetized pieces of iron at all—every block of iron at a temperature below Θ should possess, whenever it is not under the influence of an external field, the residual magnetization calculated from the intersection-point of the curve $NML(a)$ and the line of slope kT/nM which passes through the origin.

On grasping this situation, one is likely to feel that the theory has collapsed. The situation can be saved, however, by supposing that the "demagnetized" metal subdivides itself into a vast number of little regions, zones, or filaments, each of which possesses the full residual magnetism of the theory, while in direction their magnetic moments are oriented quite at random. It is not possible to identify these with individual crystals, nor with any other discernible granulations of the metal. Perhaps they are to be identified with the chains of elementary magnets once postulated by Ewing; it would be grati-

fying to make a connection between the theories of Ewing and Weiss. Perhaps they are the units from which arise the separate clicks which constitute the Barkhausen effect. As for the initial curve, attempts must be made to explain it either by supposing that the increasing field wheels the magnetic moments of the several zones gradually into parallelism with itself, or—what is more probable—that the field abruptly reverses, one after the other, all the magnetic moments which initially are inclined to it at angles superior to 90° . By suitably combining these two images, one may copy almost any possible form of initial curve. I cannot enter into these questions, except to answer as far as possible what I designated as the second question to be asked in testing the theory: what observable quantity is to be compared with the “residual magnetization” predicted from the theory of Weiss?

A piece of iron brought to saturation by a large applied field is supposed to consist of these magnetized zones, their moments all directed either parallel or at least at inclinations of less than 90° to the field. The applied field strength should elevate the magnetization of each to a value somewhat greater (corresponding to an intersection-point somewhat farther along the “curve” of Fig. 13) than the predicted “residual magnetization”; but the values of n and I and hence their product are so enormous that the addition is only slight. The saturation intensity of magnetization of the iron, $I_{\max.}$, should then be very nearly equal to the predicted residual magnetism, if all the magnetic moments are parallel; or to one half of the predicted residual magnetization, if the magnetic moments are distributed at random over the directions inclined at less than 90° to the applied field. In the former case, the variation of $I_{\max.}$ with T should follow the curve of Fig. 9; in the latter case, a curve of the same form. The actual observations upon iron, nickel, cobalt and magnetite are shown in that figure, and the reader may judge of the agreement for himself.

Comparison of Ewing's Theory with that of Langevin and Weiss

At first glance the Ewing model and the Langevin-Weiss conception of a ferromagnetic substance seem extremely different; contradictory, in fact. In Ewing's view, the perpetual effort of the applied field to align the elementary magnets is hindered by the forces which these exert on one another. In Langevin's theory, the antagonist of the applied field is the thermal agitation. Now Langevin's theory is competent to deal with paramagnetic substances which are difficult to magnetize, but not with iron and the like which are strongly mag-

netized by weak fields. This means that the thermal agitation is too strong an antagonist to the applied field. Weiss therefore provided the latter with a powerful ally, in the form of an intense molecular field parallel to it and proportional to the magnetization. The applied field and its ally together are able to overpower the thermal agitation and bring about saturation in cold iron. Now to say "molecular field" is merely to use a different phrase for "influence of the atoms on one another." In the theory of Weiss, this influence of the atoms on one another helps the field to align them; in Ewing's theory, it hinders the field. How do away with this arrant contradiction?

Perhaps a partial union may be effected, in this wise. According to Langevin and Weiss, a piece of cold iron consists of a multitude of small zones or regions of atom-groups, each magnetized to a high degree, their directions of magnetization dispersed at random; an applied field acts primarily by wheeling these magnetizations into line. According to Ewing, a piece of cold iron consists of a multitude of chains or pairs of systems of elementary magnets, which an applied field upsets, perhaps only to re-weld them anew into more favourably oriented chains. Weiss deals with the state of affairs inside the atom-groups; Ewing deals with the effect of the applied field in breaking up and rebuilding the atom-groups. Might one say that Weiss explains the conditions, under which the elementary magnets form themselves into groups or chains such as Ewing preassumed? that Ewing describes the action of the external field upon these groups, an action which Weiss left imprecise? so that the two theories, when properly revised, will complement each other? It seems possible. At all events, each of the theories has so many successes to its credit, that there can be no thought of discarding either for the sake of the other. Those who are weary of trying to reconcile waves and quanta might refresh themselves by reflecting on this problem.

McKeehan's Theory

In the theory of McKeehan, magnetostriction is promoted to the dominant role. The distortion which a metal undergoes when it is magnetized is held responsible for hysteresis, and for the fact that the rise of the *I*-vs.-*H* curve is gradual, not sudden. This view was suggested by the fact which I have mentioned already: that, in the series of the permalloys, the permeability reaches a surprisingly high maximum value and the hysteresis a surprisingly low minimum value, just at that alloy of which the magnetostriction is undetectably small until saturation is nearly attained—the alloy intermediate between

those which lengthen and those which shorten when magnetization commences. The alloy which is most rapidly magnetized when the field is gradually increased from zero, and which dissipates the smallest amount of energy when the field is varied in cyclic fashion, is also precisely the one which suffers the least deformation. From this McKeehan drew the inference, that were it not for the deformation inseparable from the act of magnetizing, the initial curve for every metal would rise swiftly from the origin to saturation, and the sides of the hysteresis-loop would fall together.

D. THE ATOMIC MAGNETS

Had I announced at the beginning of this article that some sixty pages would be spent over the data of ferromagnetism and the theories of the influence of elementary or atomic magnets on one another, and only a few closing paragraphs over the atoms which are supposedly responsible for the whole affair, the plan might have seemed most ill-adjusted to the relative interest of these divisions. Now, I hope, it will seem less perverse. The truth is, that we do not understand ferromagnetism well enough to draw from it any reliable conclusions concerning the atomic magnets. For these, we must consult the behavior of paramagnetic substances, and line-spectra, and the observations of Gerlach and Stern and their followers upon streams of atoms flying through magnetic fields.

In the apparatus of Gerlach and Stern, the atoms are probably as nearly free from mutual forces as atoms in the laboratory can ever be; having issued from a small hole in the wall of a furnace full of hot vapor, they rush swiftly across a high vacuum while they are being examined. In the mapping of absorption-spectra, the atoms are those of a rarefied gas, and are "free" in the sense in which atoms of gases are free—that is to say, they are influenced only by those agencies which establish and maintain thermal equilibrium, agencies which we commonly conceive as short, sharp collisions between atom and atom. Some paramagnetic gases behave toward an applied magnetic field as though their molecules, some salt-solutions behave as though their ions, were magnets of fixed permanent moment on which the field can act, but otherwise were free in the foregoing sense. Other gases and salt-solutions behave as though their molecules or ions were permanent magnets, influenced by the applied magnetic field and by an extra field proportional to the magnetization of the assemblage, and otherwise free except for the agencies which establish thermal equilibrium and maintain it.

In all the foregoing cases of atoms or molecules or ions enjoying

variously close approximations to perfect freedom, the theories are good enough to make it possible to bring about quantitative agreement between theory and experiment, simply by choosing appropriate values for the magnetic moments of these particles. The values so determined nearly always lie between 10^{-18} and 10^{-20} C.G.S. units.

Ferromagnetic substances are solids, and we need not be surprised that the mutual influence of the atoms becomes so great as to make the task of devising a theory much more difficult. Ewing, it is true, did show that elementary magnets of a particular shape and crowded close together would form systems displaying the peculiar features (hysteresis, and a crooked magnetization-curve) of ferromagnetics. Weiss did show that atomic magnets, subject to the agencies which bring about thermal equilibrium and maintain it, and in addition to a field proportional to the magnetization of the assemblage and enormously great, would form systems displaying residual magnetism below a certain temperature, and paramagnetic above. Dazzling as these achievements are, the theories are not so good that they can be brought into complete accord with the data, simply by choosing appropriate values for the moments of the imagined elementary magnets.

Can we at least assign a value of the order familiar among paramagnetics, 10^{-19} for instance, to the magnetic moment of (say) the iron atom—that is to say, the atoms of a piece of solid pure iron, since iron is not in all conditions ferromagnetic—without definitely contradicting any fact of experience? Probably we can. In fact, the saturation-values of the magnetizations of iron, nickel, and cobalt support this idea. If saturation signifies that all the atomic magnets are parallel, then the magnetic moment of each must be the quotient of $I_{\max.}$ by the number of atoms in unit volume; at all events, the magnetic moment of the atom cannot be less than the quotient, by that number of atoms, of the highest value of I ever observed. Now the highest values of I are observed at the lowest temperatures; extrapolating from the data (shown in Figure 9) to zero absolute, Weiss obtained values of the quotient which are indeed of the order 10^{-19} —eleven “magnetons” for iron and three for nickel, and probably eight for cobalt. This concordance with the values of magnetic moment to which we are accustomed among free atoms is evidently important. However, as Ewing found, we cannot take the natural next step of supposing that each atom is a long slender magnet having its ends very close to the ends of the adjacent magnets; for then the I -vs.- H curve of the assemblage would not agree with the initial curves observed in practice.

Everyone now agrees with the idea, proposed more than a century ago by Ampère, that atoms are magnets because of the circulating charges which they contain. The estimates of atomic moments deduced from line-spectra are based on this assumption, and the verified correctness of these estimates sustains it. Now, if a magnetic atom is a whirl of electricity, it possesses angular momentum as well as magnetic moment. If so, the process of magnetizing an iron wire involves the bringing-into-parallelism of myriads of spinning-tops, of which the angular momenta when all aligned combine into a respectable sum. If this goes on inside a wire during magnetization, there should be a "recoil" somewhere, comparable to the recoil of a gun when a shell is fired—the suspension of the wire should receive an opposite angular momentum, experience a torque. Conversely, the process of twisting an unmagnetized wire should impress a lateral torque upon myriads of spinning-tops of which the axes point in directions scattered at random; each of these should be urged to set itself more nearly parallel to the axis of the twist, which is the axis of the wire; and the twisting should therefore magnetize the wire.

Both of these effects, which jointly are called the "gyromagnetic effect," have been detected and measured. From the measurements (thus far performed upon iron, nickel, cobalt, magnetite and a Heusler alloy), it results that the ratio of the angular momentum P to the magnetic moment M of an elementary magnet conforms to the equation:

$$P/M = mc/e,$$

in which m stands for the mass of the electron and e/c for its charge measured in electromagnetic units. *This is the value which would be expected for the ratio, if the elementary magnet is an electron spinning upon itself.*

Now there are weighty reasons for supposing that the conception of a "spinning electron," possessing a fixed characteristic angular momentum and a permanent magnetic moment e/mc times as great, may be what is required to complete the theory of line-spectra of free atoms which Bohr began. The gyromagnetic effect of the ferromagnetic solids therefore indicates that the elementary magnets scattered through these are the same as the elementary magnets located in free atoms—they are electrons, or groups of electrons suitably linked together. The test cannot be made upon paramagnetics, for they cannot be (or at least have not yet been) strongly enough magnetized. Ferromagnetic substances are the only ones which in a feasible field acquire so great a magnetization that the

recoil from the spinning electrons is detectable. This seems to be as yet the only contribution of ferromagnetism to contemporary atomic theory.

Yet even if we take it for settled that the elementary magnets within the atoms of a solid piece of iron are spinning electrons, the real problem of ferromagnetism remains unsolved. If the elementary magnets in iron are just like those in all other atoms, how does it happen that iron and two other elements alone may be ferromagnetic? that even iron may cease to be ferromagnetic, if mixed with a little manganese? that manganese and copper and aluminium can become ferromagnetic when and only when alloyed together? Since apparently we must not suppose that each atom of iron is distinguished from all those of never-ferromagnetic substances through having a peculiar kind of magnet inside it, we must suppose that something strange in the arrangement of the electron-magnets of the iron atom permits it to be so distorted, and so to distort its neighbors, that on occasion its neighbors and itself jointly develop ferromagnetism. There is something extraordinary about the systems of 26 and 27 and 28 electrons about a nucleus, which iron and nickel and cobalt atoms are. Their individual electrons are not unique; by themselves, or as ions in a solution, they show nothing unique; but they turn into something unique when they are rightly compounded together into a solid. The theories of ferromagnetism and the gyromagnetic effect have limited without solving the fundamental problem of ferromagnetism: what is it that makes the difference between the ferromagnetic substances, and all the rest?

ACKNOWLEDGMENTS AND REFERENCES

The foregoing article is based largely upon the books of J. A. Ewing (*Magnetic Induction in Iron and Other Metals*; Electrician, 1900), P. Weiss and E. Foex (*Le Magnétisme*; Colin, 1926) and E. C. Stoner (*Magnetism and Atomic Structure*; Methuen, 1926); the articles by S. Bidwell in the eleventh edition of the *Encyclopædia Britannica*, by P. Debye in volume 6 of the *Handbuch der Radiologie*, by E. Gumlich and R. Gans in *Die Kultur der Gegenwart*, by K. Honda in the *Dictionary of Applied Physics*; and the articles of L. W. McKeehan on ferromagnetism (*Journ. Franklin Inst.* **197**, pp. 583-602, 757-786; 1924), magnetostriction (*ibid.* **202**, pp. 737-773; 1926) and the permalloys (*Phys. Rev.* (2) **28**, pp. 146-166; 1926, and others there cited).

Some of the very recent papers upon magnetization of single crystals are those of W. L. Webster (*Proc. Roy. Soc.* **A107**, pp. 496-509; 1925); K. Honda and S. Kaya (*Tohoku Univ. Sci. Rep.* **15**, pp. 721-753; 1926); W. Gerlach (*ZS. f. Phys.* **38**, pp. 828-840; 1926). For magnetostriction of single crystals, see W. L. Webster (*Proc. Roy. Soc.* **A109**, pp. 570-584; 1925) and K. Honda and Y. Mashiyama (*Tohoku Univ. Sci. Rep.* **15**, pp. 755-776; 1926). For the data concerning permalloys see,

in addition to the papers already cited, that of H. D. Arnold and G. W. Elmen (*Journ. Franklin Inst.* **195**, pp. 621-632; 1923). The gyromagnetic effect, the data and the theories of paramagnetic substances, and diamagnetism are treated very fully in the above-cited book of Stoner; paramagnetism, and the interesting and important magneto-caloric effects which I had not space to discuss, in the book of Weiss and Foex; diamagnetism in a late article by E. S. Bieler (*Journ. Franklin Inst.* **203**, pp. 211-242; 1927).

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